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THE EVAPORATION PROCESS AND ITS APPLICATION TO THE ALUMINIZING OF LARGE TELESCOPE MIRRORS

JOHN STRONG

ABSTRACT

A history of the evaporation process is given. This includes the contributions by Ritschl and others and describes the present technique in detail.

Formulae are developed which give the thickness of the film produced by a circular array of evaporation sources. Applications for the cases of 40-inch and the 108-inch tanks are discussed. Reference is also made to the application of non-uniform films in the figuring of mirrors. Methods of cleaning mirrors preparatory to coating are discussed. The technique of obtaining high vacuum in large tanks is treated.

The reflectivities and other properties are given for evaporated films of aluminum and silver, as well as a *Cr*, *Pt*, *Pd*, *Rh*, *Sn*, *Au*, and *Cu*. This includes observations on aluminized astronomical mirrors now in use for over three years.

The results of a study of the oxidation (corrosion) of aluminum by means of measurements on transmissivity and reflectivity of partial films is reported.

The different sets of equipment that have been developed in increasing size up to the 108-inch tank are described.

The history of the application of the process to astronomical mirrors is given.

INTRODUCTION

A process for coating glass mirrors with a reflecting material is described. It involves the evaporation of the material (usually a metal) in a high vacuum and is therefore called the "evaporation process." The metal is heated until it has a vapor pressure of the order of 1/10 mm of mercury. The vacuum is sufficiently high to give a long free path (greater than 100 inches), so that the evaporated molecules travel without intermolecular collisions from the hot metal source to the cold surface of the mirror. The attainment of

this vacuum during evaporation is evidenced by the sharp shadows cast in the evaporated film by obstacles placed between it and the evaporation source. The remarkable feature of the whole phenomenon is that the film, condensed on the mirror, has the same degree of polish as that possessed by the glass. A mirror freshly coated by this process does not, therefore, require any burnishing.

C. Hawley Cartwright and the author studied the application of this process to many metals, including aluminum, but it was not until the technique for aluminum was developed by the author that films of high quality were obtained which showed its potential value for coating astronomical mirrors.

The application to very large mirrors seemed to offer some difficulty, especially in the attainment of the necessary vacuum. Recent advances in vacuum technique and, in particular, the advent of oil diffusion pumps, have helped to make this application possible.

HISTORY OF THE EVAPORATION PROCESS

Pringsheim and Pohl[1] discovered that several metals, including aluminum, could be evaporated in vacuum and condensed on a glass surface to form a polished reflecting film. They used a magnesia crucible from which to distil the metal. R. Ritschl,[2] in 1928, in making an application of the evaporation method to the preparation of half-silvered interferometer mirrors, heated the metal in a bare tungsten coil. This change in technique has the advantage over the use of a magnesia crucible that the tungsten does not evaporate or out-gas so much in a vacuum.

Since then several physicists have used this technique for making mirrors for experimental use. Those with whom the author is acquainted are A. H. Pfund,[3] Joseph E. Henderson,[4] D. L. Webster,[5] W. W. Nicholas,[6] H. C. Burger and P. H. Van Cittert,[7] P. G. Kruger,

[1] *Verh. d. Deut. Phys. Ges.*, **14**, 506, 1912.

[2] *Tätigkeitsbericht d. Phys. Techn. Reichsanstalt*, 1928; *Zs. f. Phys.*, **69**, 578, 1931.

[3] *Rev. Sc. Instr.*, **1**, 397, 1930.

[4] *Evaporation of Nickel from Tungsten*, Yale University, 1926.

[5] *Proc. Nat. Acad. Sci.*, **14**, 679, 1928.

[6] *Circ. Bur. of Standards*, No. 389.

[7] *Zs. f. Phys.*, **66**, 218, 1930.

Robley C. Williams,[8-9] George B. Sabine,[10] C. Hawley Cartwright,[11-12] and Hiram W. Edwards.[13]

The author began his experiments at the University of Michigan in 1929, starting with a technique described by Professor L. S. Ornstein during his visit there. The experiments, originally started by Professor E. F. Barker and later carried on by the author, included not only the evaporation of metals but also their protection from tarnish by a thin film of evaporated quartz. This distillation of a non-conductor is a possibility of the evaporation process not shared by the sputtering method. Films of quartz, as well as of *AgCl*, were applied by this method to alkali halide prisms, used in infra-red spectroscopy, to prevent them from tarnishing with moisture. Silver chloride was found to give the best protection for these prisms.

Cartwright and Strong,[11] at the California Institute of Technology, in the autumn of 1930 developed a simple apparatus for carrying out the process and made a survey of its applicability to different metals. The usual technique of heating some of the metal to be evaporated in a helix of tungsten wire was found to be successful, except with aluminum and beryllium. They observed that the tungsten coil was dissolved by these metals. A similar failure for aluminum was recorded by W. W. Nicholas in the Bureau of Standards circular, *The Making of Mirrors by the Deposition of Metal on Glass.*

EVAPORATION TECHNIQUE FOR ALUMINUM

Other attempts[12] have since been made to develop the technique of evaporating aluminum because of its high ultra-violet reflectivity. Experiments were carried out here with crucibles of graphite, pure fused magnesia and alumina (sapphire), as well as with sintered and with fused crucibles of thorium oxide. They showed that the heating in a crucible was apparently impractical; either the metal reacted chemically with the material of the crucible or the latter evaporated when the aluminum was heated. The latter crucible material was

[8] *Phys. Rev.*, **41**, 255, 1932.
[9] *Ibid.*, **46**, 146, 1934.
[10] *Ap. J.*, **77**, 316, 1933.
[11] *Rev. Sc. Instr.*, **2**, 189, 1931.
[12] *Ibid.*, **3**, 298, 1932.
[13] *Phys. Rev.*, **43**, 205, 1933. *Rev. Sc. Instr.*, **4**, 449, 1933.

prepared by a method described by W. H. Swanger and F. R. Caldwell.[14]

The bare tungsten-coil method, however, proved to be the most practical after the discovery by the author that tungsten has a limited solubility in molten aluminum.[15] The burning-out of the tungsten coil was avoided by the simple expedient of making it of larger wire and arranging the charge so that the solubility of the molten aluminum for tungsten could be satisfied without dangerously reducing the diameter of the wire. A chemical analysis of the tungsten alloy formed when aluminum is fused on a tungsten coil showed the solubility of tungsten in aluminum to be about 3 per cent by volume.

It might be expected that some of the dissolved tungsten would boil away, especially since its spectrum has been observed in the bell jar during evaporation.[16] In order to test this point a coil was weighed before and after evaporating several charges of aluminum. Instead of a loss in weight, an increase was observed, indicating that some aluminum had diffused into the tungsten. Extended firing at a very high temperature decreased the weight, however, until, within the experimental error, it became the same as in the beginning. A chemical analysis of the condensed metal film gave no definite indication of tungsten. A concentration of 0.03 per cent by weight was judged detectable by Dr. T. F. Anderson, to whom I am indebted for the chemical analysis. He found 0.2 per cent of iron as the chief impurity of the film. The tungsten which is dissolved thus appears to be almost completely precipitated back on the coil as the evaporation proceeds. It may not deposit in exactly the same place, but it does compensate in a large measure for the decrease in diameter of the tungsten wire, especially if the coil is turned over after each charge. Ordinarily one coil lasts for the evaporation of about a dozen charges.

The final arrangement of the coil, which has been used for aluminizing all mirrors at the California Institute of Technology, is shown in Plate XIV. It is in the form of a helix, consisting of ten turns of 30-mil tungsten wire, $\frac{5}{16}$ inch in diameter and pitched four turns to the inch. A U-shaped piece of aluminum wire 1 mm in diameter and

[14] *Bur. of Standards J. of Res.*, **6**, 1131, 1931.

[15] John Strong, *Phys. Rev.*, **43**, 498, 1933.

[16] E. Gaviola and John Strong, *ibid.*, **38**, 136, 1935.

PLATE XIV

Aluminum Coils Showing Method of Charging and Melting Down the Charge

about 10 mm in total length is clamped to each turn, as is shown by the top coil in the illustration. A potential of 20 volts applied to the coil in vacuum for four seconds fuses these pieces, as is shown by the coil in the middle. At this stage surface tension keeps the molten aluminum from dropping. This fusion also serves to free it from oxide and other impurities. It is customary to make a separate run in order to effect this melting of the aluminum to the tungsten wires in all the tanks except the 40-inch. Here the coils are covered by a baffle during the preliminary firing. The aluminum is distilled from the coils by the same voltage applied to each coil for fifteen seconds, leaving the coil as is shown at the bottom of the illustration.

UNIFORM FILMS

In order to get a uniform coat on the mirror, it was early decided to distil the metal on the large reflector surface from several tungsten sources suitably arranged, rather than from one movable source.

The evaporation of polonium in a high vacuum from a point source has been investigated by Bonet-Maury.[17] He finds that the condensation on a plane surface is proportional to the inverse square of the distance from the source and to the cosine of the angle between the normal to the surface and the line connecting the surface with the source. We have no reason to suspect that such is not the case for other metals which have a low vapor pressure at room temperatures.

Starting with this, we may consider the distribution of the film thickness τ produced by various experimental arrangements. In the case of evaporation to the inside surface of a sphere of radius ρ from a point source of vapor at its center we get a uniform film whose thickness, τ_0, is

$$\tau_0 = \frac{m}{4\pi\delta\rho^2} .$$

Here m is the mass of metal evaporated and δ is its density.

The film thickness on a plane surface at the normal distance ρ from a point source at P is

$$\tau_P = \frac{m}{4\pi\delta r^2} \cos\theta = \tau_0 \left(\frac{\rho}{r}\right)^3 .$$

[17] *Ann. d. Phys.*, **11**, 285, 1929.

Here τ_0 is the thickness at distance ρ, r is the distance from the source to P, and θ is the inclination of the surface at P to the molecular rays emitted by the source.

The film thickness produced on a plane surface by a circular array of vapor sources was studied. If there are N coils spaced uniformly around a circle at a distance ρ from the surface to be coated, the film thickness at P, which is at a distance a from the intersection of the axis of the circle with the face of the mirror, is given by the expression

$$\tau_P = \frac{M\rho}{4\pi\delta N} \sum_{i=1}^{N} \frac{1}{r^3} .$$

Here M is the total mass of metal evaporated and r is the distance from P to the respective ith coil.

Dr. Edward M. Thorndike made the same calculation, assuming a continuous circular source. The thickness is given in this case by

$$\tau_P = \frac{M\rho}{8\pi^2\delta} \int_0^{2\pi} \frac{d\theta}{r^3} .$$

This calculation involves the integration[18] of the expression

$$\int_0^{2\pi} \frac{d\theta}{r^3} = \int_0^{2\pi} \frac{d\theta}{(1+a^2+\rho^2-2a\cos\theta)^{3/2}}$$

$$= \frac{4}{[(a-1)^2+\rho^2]\sqrt{(a+1)^2+\rho^2}} E\left(\frac{2\sqrt{a}}{\sqrt{(a+1)^2+\rho^2}}\right) .$$

θ defines the position of the element $d\theta$ of the circular source involved in the integration. Values of this integral, calculated by Thorndike, are given in Table I.

We see from this table that for $\rho = 1$ the film is quite uniform as far out from the center as $a = 1$. This case was realized in the 40-inch tank by a circular array of twelve of the standard coils (Pl. XIV) spaced around a circle 36 inches in diameter, 18 inches above the face of the astronomical reflector to be coated (Fig. 1). Tests of

[18] D. Bierens de Haan, *Nouvelles tables d'intégrales définies*, Table 67, eq. 3, p. 102.

TABLE I*

VALUES OF THE INTEGRAL $\int_0^{2\pi} \frac{d\theta}{r^3}$ FOR VARIOUS PARAMETERS

a	$\rho=0.5$	$\rho=1.0$	$\rho=1.1$	$\rho=1.2$	$\rho=2.0$	$\rho=4.0$
0.00	4.50	2.22	1.91	1.65	.560	.090
0.25	4.82	2.24	1.93	1.65	.555	.090
0.50	3.96	2.29	1.93	1.63	.540	.088
0.75	7.74	2.28	1.89	1.57	.515	.085
0.80	....	2.27	....	....	....	....
0.90	....	2.22	....	....	....	....
1.00	8.28	2.11	1.74	1.45	.480	.082
1.50	3.40	1.38	1.09	1.02	.385	.072
2.00	1.20	0.74	0.67	0.61	.285	.068
3.00	0.28	0.24	0.23	0.22	.145	.050

* a and ρ are expressed relative to the radius of the source taken as unity.

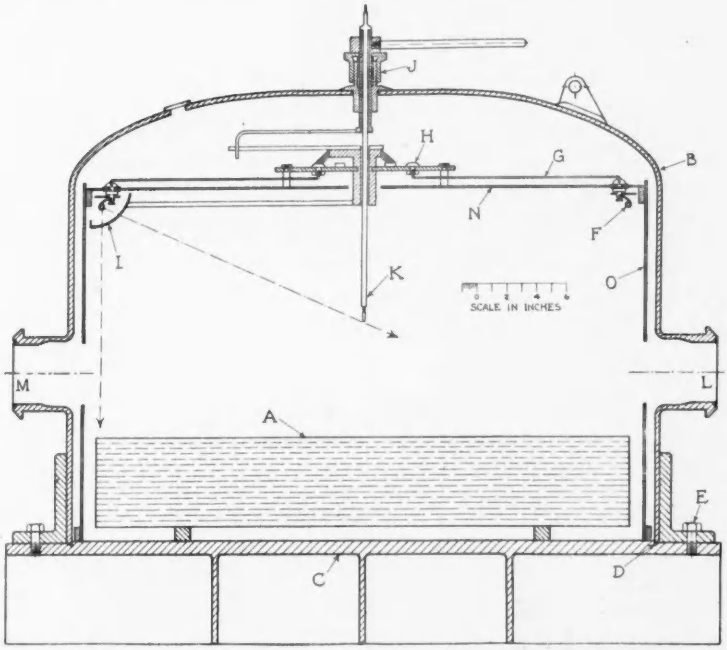


FIG. 1.—Diagram of apparatus for aluminizing mirror. *A*, mirror to be aluminized (36-inch diameter); *B*, bell-jar ($39\frac{1}{2}$ in. inside diameter); *C*, base plate; *D*, groove for lead fuse wire gasket; *E*, bolts to fasten bell-jar to base plate; *F*, filaments of tungsten wire from which aluminum is evaporated; *G*, conductors to supply current to filaments; *H*, switch; *I*, baffle; *J*, packing gland for switch control; *K*, electrode for cleaning the mirror face; *L*, observation window; *M*, pump connection; *N*, plate to carry filaments and switch; *O*, removable brass cylinder to carry *N*.

transmission of a film produced with partially loaded coils showed that the coat had the expected uniformity.

In the 108-inch tank it was not convenient to use a similar array of coils spaced 50 inches from the face of the mirror. Instead, three arrays were used, each about 20 inches from the mirror. From the expressions developed above, as well as from actual tests, it was found that with four coils in the center, twelve on a circle of 50-inch diameter, and twenty-four on a circle of 100-inch diameter, the film would have sufficient uniformity. The arrangement is shown in Plate XV.

The foregoing arrangement produced a film which was just opaque to sunlight. It was desirable to have this thickness (about 1000 A) since much thicker films are more easily scratched, while thinner ones may in time become transparent because of the gradual increase in thickness of the oxide layer which forms on the aluminum coat.

NON-UNIFORM FILMS

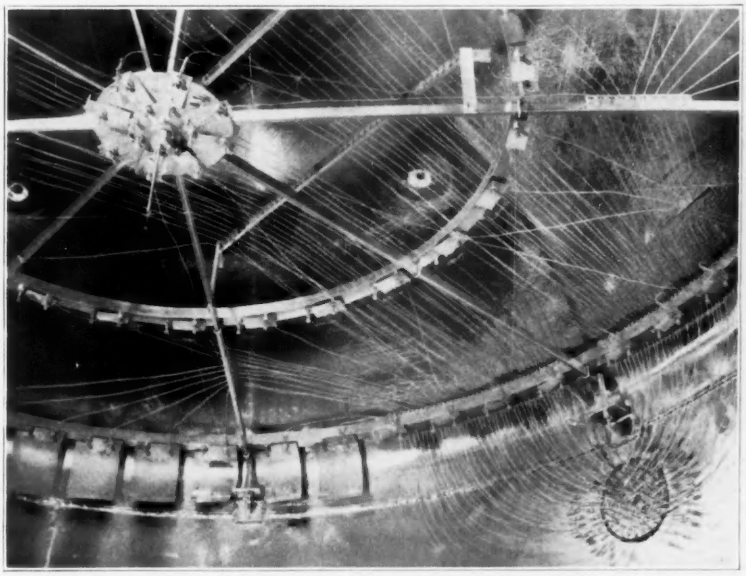
The film may be made non-uniform in thickness as desired, by a suitable arrangement of the evaporation coils or by the use of baffles, thus allowing the figure of a mirror to be altered. It is undesirable for the mirror to be refigured every time that it is given a new reflecting coat. Utilizing the insoluble nature of chromium,[25] we have devised a scheme whereby the mirror is figured with chromium. A uniform reflecting coat of aluminum may then be added or removed without affecting the true surface given by the chromium film. John Strong and E. Gaviola[19] have reported on the successful parabolizing of a 12-inch $f/7$ spherical mirror. This technique is described in an article appearing in the April, 1936, issue of the Journal of the Optical Society of America.

CLEANING THE GLASS

It is important that the surfaces to be aluminized be clean. Difficulty is met in freeing it from the contamination which is usually deposited by cotton used for drying and which is believed to be a film of fatty acid. If it is not removed, it reacts with the aluminum;

[19] Meeting of the Astronomical Section of the American Association for the Advancement of Science, June, 1935.

PLATE XV



Inside View of the 108-Inch Tank, Showing Electrical Leads at Lower Right, Outer Array of 48 Coils, Intermediate Array of 24 Coils, and Central Array of 8 Coils and Electrode with Attached Aerial for Electrical Cleaning Discharge

The tank was lined with Apiezon wax-*W*

a completed mirror which at first looks perfect will develop, in a day or two, countless tiny blisters. Even if these blisters do not develop, the adhesion of the aluminum is weakened. In former papers [20-21] a procedure was described for removing this film by a glow discharge.

The glass surface to be aluminized is cleaned by washing it with the detergent, Alphasol, then with *KOH*, and finally with concentrated HNO_3. The mirror is rinsed between each washing, with tap water. This order of the reagents is the reverse of that used in silvering because it has been observed that fatty acids adhere to the alkaline glass surface more readily than to the acidified glass surface.[22] After washing with tap water and finally with distilled water, the surface is dried and rubbed with absorbent cotton. The breath should condense on the glass, after this treatment, to form a uniform structureless gray film. If a pattern persists in the breath figure, it may be removed by a light rubbing with a rouge pad, after which the foregoing cleaning process is repeated. Small mirrors which cannot be conveniently held for drying without contamination from the fingers may be allowed to drain and dry by evaporation. Even though the breath figures in this case may have some structure, the use of the glow discharge will usually result in an excellent mirror. Very small mirrors may be simply treated in a beaker with the various reagents, rinsed and arranged to dry on a tuft of absorbent cotton.

The Alphasol[23] referred to above is one of the new detergents,[24] all of which have in common the constitution of sulphonated organic compounds of high molecular weight. These detergents have a neutral reaction and their advantage over soap lies in the fact that they may be used in neutral, caustic, or even acid solutions. Unlike soaps, the new detergents form soluble compounds with the magnesium and calcium salts common in tap water, and therefore rinse easily.

[20] John Strong, *Pub. Astr. Soc. of Pac.*, **46**, 18, 1934.

[21] John Strong, *Rev. Sc. Instr.*, **6**, 97, 1935.

[22] See also Katharine B. Blodgett, *J. Amer. Chem. Soc.*, **57**, 1007, 1935.

[23] The compound Alphasol *O T* is manufactured by the Selden Division of the American Cyanamid and Chemical Corp., Bridgeville (Pittsburgh), Pa.

[24] R. A. Duncan, *J. Ind. and Eng. Chem.*, **26**, 24, 1934.

The cleaning of astronomical mirrors is much more difficult than the cleaning of small pieces of plate glass, because of the rouge and oxidized pitch in the rough ground edges as well as in bubble holes which are often present in the face of the mirror. It is difficult to avoid spreading such contamination when the mirror is rubbed with cotton.

The glow discharge necessary for the final cleaning of the mirror is conveniently produced throughout the inside of the tank by a spark coil, or a 1-kw transformer in the case of the 108-inch tank. The discharge is produced by an electrode, or an aerial in the case of the 108-inch tank (Fig. 1 and Pl. XV).

One advantage of the chroluminum process developed by Williams is that it facilitates the aluminizing of mirrors which are otherwise difficult to clean. This can also be attained with double films where the underlying one is not chromium. The chromium, however, exhibits strong adhesion to glass,[25] without the extreme cleanliness that is required for good adhesion with aluminum. Furthermore, the chromium film is very hard. The aluminum adheres strongly to the freshly formed chromium surface. It is not, however, necessary to coat the mirrors with a double film if the cleaning technique described above is employed.

After installation in the telescope it is advisable to wash aluminized mirrors from time to time to remove dust and other contamination which accumulates with use. This is done with Alphasol, as this detergent may be rinsed off with tap water without leaving any stain. Surface film not removed by this washing will frequently dissolve in concentrated nitric acid. Sometimes it is possible to remove faint bloom with a rouge pad such as is commonly used to burnish silvered mirrors.

VACUUM TECHNIQUE

The first apparatus constructed for the application of the evaporation process was equipped with high-speed mercury pumps. This

[25] It has been discovered here that chromium films may be removed with cold hydrochloric acid if a little zinc dust is first sprinkled on the metal, otherwise it is not easy to remove them. Another technique involves making the film cathode in concentrated *HCl*. Once the solution is started, it will proceed regularly over the surface of the mirror.

will be referred to later. Cartwright and Strong later recommended charcoal absorption since, unlike most other cases where high vacuum is required, the absorption capacity of charcoal is entirely adequate for the short time involved.

One plan for evacuating the 40-inch tank involved the use of the charcoal absorption trap shown in Figure 2. The charcoal was outgassed with the heater unit shown in the figure. The water-cooling coils dissipated the heat radiated by the charcoal. After this the heater units were withdrawn; the charcoal was cooled to room temperature with tap water and then to a very low temperature with liquid air. About $2\frac{1}{2}$ pounds of liquid air were used in making one evaporation with the 40-inch tank. The vacuum attained, although not as good as that obtained with diffusion pumps, was sufficient for carrying out the evaporation. A rough estimate of the pumping speed of this trap indicated a value for air of about 30 liters per second for several minutes. However, the method was abandoned in favor of Apiezon-oil diffusion pumps, because they have a high pumping speed for hydrogen, one of the gases evolved during evaporation.

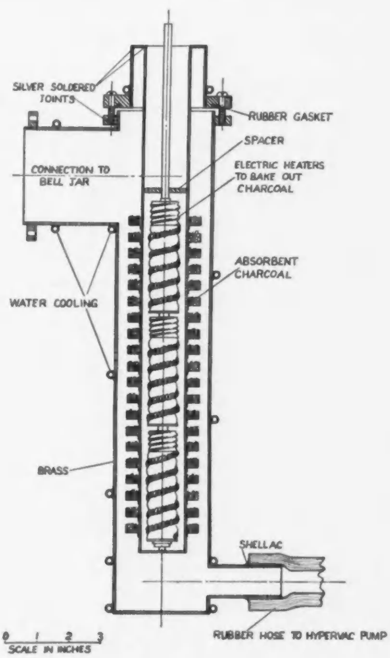


FIG. 2.—Charcoal absorption trap for obtaining high vacuum.

The Apiezon-oil pumping systems will be treated in a separate paper to be published in the *Review of Scientific Instruments*. It will be sufficient here to describe the general features of the pumping system for the 108-inch tank.

The original plan called for a charcoal trap with this pumping system, similar to the one described above, which could be cooled with "dry ice." However, it was not found necessary to use it. A long

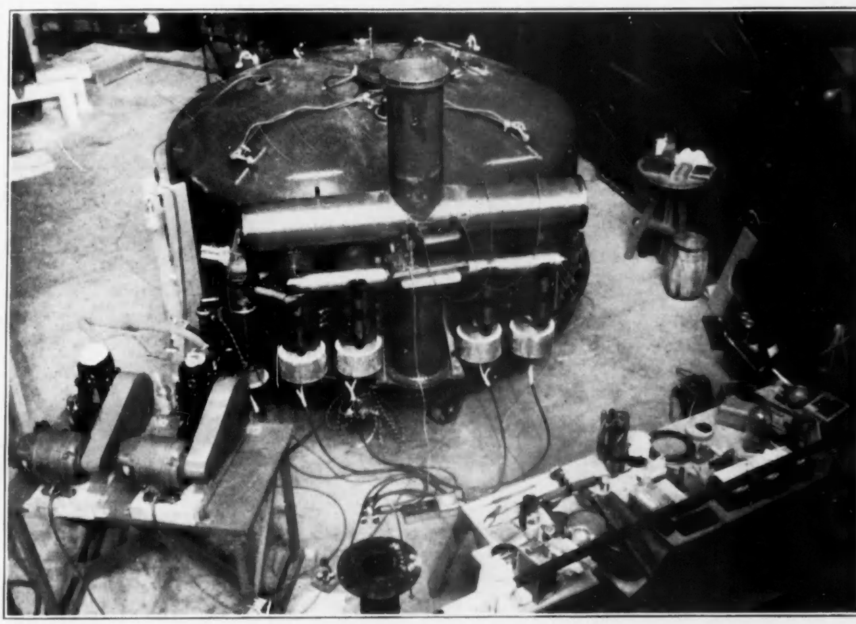
vertical pipe (Pl. XVI) was constructed to contain this trap. This pipe was connected with the 108-inch tank as well as with the horizontal manifold which supports the diffusion pumps. The manifold was connected to the three 8-inch diffusion pumps on the right, which discharge into a chamber above the 8-inch pump on the left through two manifolds, one in front and the other behind the pumps. This pump is, in turn, backed by a 6-inch and a $2\frac{1}{2}$-inch pump. Two Hypervac mechanical pumps serve to maintain the fore-vacuum for these diffusion pumps. They were capable of evacuating the tank to a pressure of 10^{-2} mm in approximately eight hours. The glass discharge tubes for testing the vacuum were connected, respectively, with the inlet and the exhaust of the last stage of the pumping system, i.e., with the three 8-inch pumps at the right, which were arranged in parallel connections.

After the tank is sealed the mechanical pumps are started. In about eight hours they lower the pressure to a point at which the diffusion pumps may be turned on. When the diffusion pumps are hot, about three-quarters of an hour after the current is switched on, they begin to pump. This is shown very clearly by the discharge tubes. The tube to the right has a greater conductivity and shows a stronger discharge than the left one when they are both at the same pressure. When pumping starts, the discharge at first equalizes and finally disappears completely in the right tube. Soon after, the discharge also disappears in the other tube.

The purpose of this rather elaborate pumping system is not so much to realize a degree of vacuum at which the evaporation process can be performed, for one hypervac pump gives a vacuum of 5×10^{-4} mm of mercury in the 108-inch tank. Its purpose is the maintenance of a high vacuum after evaporation is started, in spite of the outgassing induced by the lighted filaments. The aluminizing process has a tendency to improve the vacuum by the "getter" action, and if the evaporation can be done rapidly enough, the pumping speed required can be materially reduced. Another advantage is that with such fast pumps the time involved in evacuation is less, and also considerable leaks may be tolerated. It is planned to use the same pumping system for evacuating the 200-inch tank when it is built.

The performance of this pumping system is illustrated by the log

PLATE XVI



Vacuum Pumping System and General View of Installation in the 100-Inch Telescope Dome

of the run in which the 100-inch mirror was aluminized. This is given in Table II.

It was found useful to line the 40-inch and 108-inch tanks with Apiezon wax "W," which has a vapor pressure so low that it is possible to evaporate aluminum directly on it.[26] It stops all leaks and covers up such "dead alleys" as are common in steel, so that the tank has, in effect, the strength and convenience of steel and the vacuum behavior of a glass tank. It is believed that this technique of coating the inside with Apiezon wax may be of value elsewhere in

TABLE II

LOG OF THE VACUUM WHEN THE 100-INCH MIRROR WAS ALUMINIZED

8:10 P.M.	Pumping started with two Hypervac pumps.
7:55 A.M.	Vacuum 5×10^{-3} mm mercury
	High-tension discharge from kilowatt transformer for 15 minutes at 6×10^{-2} mm of mercury air pressure
8:10 A.M.	Diffusion pumps turned on
9:33 A.M.	Vacuum 2×10^{-5} mm of mercury
	Mirror aluminized
	40 coils fired 15 seconds each
10:45 A.M.	After aluminizing
	Vacuum 5×10^{-5} mm of mercury
	Mean free path about 150 inches
	Diffusion pumps turned off
2:00 P.M.	Vacuum 3×10^{-3} mm of mercury
	Air introduced and mirror removed from tank

obtaining high vacua in large steel tanks. When first evacuated with a single Hypervac pump, a pressure of 5×10^{-4} mm of mercury was attained in the tank, indicating that there were no significant leaks. After this it was coated with two coats of Glyptal on the outside, one red and one machine blue, to insure complete coverage.

REFLECTIVITIES AND PROPERTIES OF EVAPORATED FILMS

Reflectivity measurements have been made on several films with monochromatic light obtained with a Müller-Hilger double-quartz monochromator. A Nernst filament as well as the sun served as light-source, and quartz sodium and potassium photo-cells as receivers. The photocurrent was measured with an electrometer by a

[26] At 180° C the vapor pressure of this compound is 10^{-3} mm of mercury.

null-deflection method, as well as with an amplifier and galvanometer. In every case the square of the reflectivity was measured, thus giving a higher precision in the result. The probable error of the measurements is smaller than 1 per cent for R greater than 85 per cent, and is about 2 per cent elsewhere.

The results of the measurements on various films are shown in Figures 3, 4, and 5. They include measurements on silver films which have tarnished for two and three weeks and also films which have been burnished.

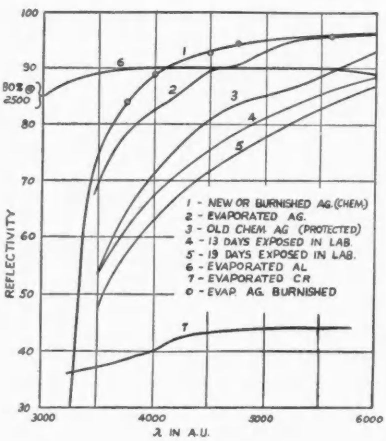


FIG. 3.—Reflectivity measurements made with the Müller-Hilger double spectrometer-monochromator, Nernst filament light-source, sodium-quartz photocell and electrometer.

W. W. Coblentz and R. Stair[27] have recently measured the reflectivity of fresh silver, and have obtained results which are definitely higher than the values obtained by Hagen and Rubens.[28] The author's measurements are in agreement with those of Coblentz and Stair in indicating definitely higher reflectivity for fresh silver than that obtained by others.

It is important to point out that differences were observed for gold, copper, and silver between evaporated and polished massive metal. This is shown, in the case of silver, where burnishing with a rouge pad increases the reflectivity from that characteristic for the evaporated metal (curve 2, Fig. 3) to that characteristic for burnished "chemical" silver (points on curve 1). Freshly silvered mirrors (by the chemical method) are frequently "toughened" by rubbing them with cotton before they are burnished with a rouge pad. Otherwise they exhibit a faint reddish color, indicating that they are charged with rouge. It is probable that both chemical and evaporated silver require burnishing to form the metal into a compact film

[27] *Bur. of Standards J. of Res.*, **2**, 343, 1929.

[28] *Ann d. Phys.*, **1**, 352, 1900; **8**, 1, 1902.

which exhibits the higher reflection coefficient. Figures 3 and 4 show also the reflectivities of *Cr*, *Pt*, *Pd*, *Rh*, and *Sn*.

The reflectivity of a 12-inch telescope mirror coated in October, 1932, and used continuously since that time, was recently tested by the substitution method by comparing it with a freshly aluminized mirror of equal radius of curvature. An incandescent lamp filtered with A, B, and C Wratten filters, respectively, was used as the light-source for these tests. It showed no decrease of reflectivity in the red and green and a decrease of 1 per cent in the blue.

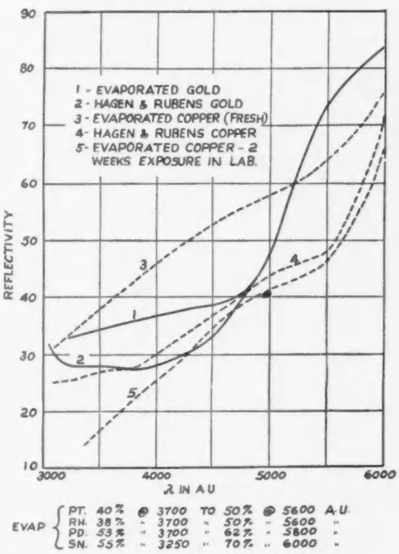


Fig. 4.—Reflectivity measurements made with the Müller-Hilger double spectrometer-monochromator, Nernst filament light-source, sodium-quartz photocell and electrometer.

The superior reflectivity of aluminum over silver for short wave-lengths not only gives a more even exposure of the stellar spectra in the violet but extends the range of the observable spectrum some 250 A, making observations possible to the limit of the transmission of the atmosphere. For the demonstration of the excellence of aluminum mirrors for photographing stellar spectra in the ultra-violet we are indebted to the pioneer work of Williams and Sabine, and later of Williams and Boothroyd, as well as to the work of W. H. Wright.

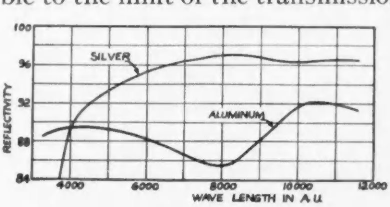


Fig. 5.—Reflectivity measurements made with the Müller-Hilger double spectrometer-monochromator, potassium photocell with amplifier and galvanometer. Sunlight used as light-source.

Figure 5 shows that aluminum is inferior to silver in reflectivity in the red and near infra-red. For spectroscopic studies in this spectral range where the light is weak this is serious. It is suggested that for this work the aluminized mirror may be temporarily coated with

silver by the chemical process. Afterward it may be removed with concentrated nitric acid, leaving the aluminum unaffected.

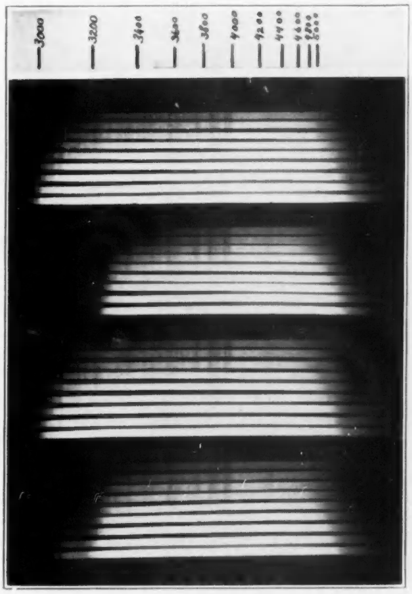
In this connection the photographs of the sky spectrum shown in Plate XVII are of interest. The spectral distribution here corresponds to that of a hot star. These photographs were taken with a Hilger E-30 quartz spectrograph on Eastman 40 plates. As the slits were 0.0055 inch in width and the dispersion at H and K was only 11 A/mm, the absorption lines are not very distinct. Nevertheless, the features of these spectra as they are influenced by reflection from silver and aluminum are evident. The exposure times within each of the four series were increased by factors of 2, the shortest being 4 seconds. The top series was exposed to the zenith at 2:00 P.M. on July 30, 1935, with three reflections from aluminum at an incidence of 45°. The second series was taken at 2:30 P.M. with three silver reflections. The two series at the bottom were taken at 4:00 and 4:30 P.M. with one aluminum and one silver reflection, respectively. After account is taken of a gradual decrease in the strength of the light-source during the afternoon, it is seen that the blackening beyond 5000 A is sensibly the same in all four series. The gain of aluminum over silver in the violet and ultra-violet is greater in the case of three mirrors, the extension of the spectrum by aluminum over silver being about 250 A.

This increase in the length of the spectrum is also shown by the photographs (Pl. XVII) of a planetary nebula taken by W. H. Wright with the Crossley reflector before and after aluminizing. Dr. Wright[29] states that photographs of the north polar sequence taken by Dr. Mayall before and after the mirror was aluminized show a limiting magnitude of 19.3 with silver and of 19.8 with aluminum. Tests made after one year of service did not indicate any change in this limit.

If we choose one of the various reflectivity-curves for silver shown in Figure 3, in such a way that aluminum gives a gain of $0^{M}5$ (photographic) for an A0 star, we compute that the gain for a Cassegrain arrangement with three reflections is about 1 mag. This calculation is made for an Eastman 40 plate and for the normal transmission of the atmosphere. It will be of interest to see how this cal-

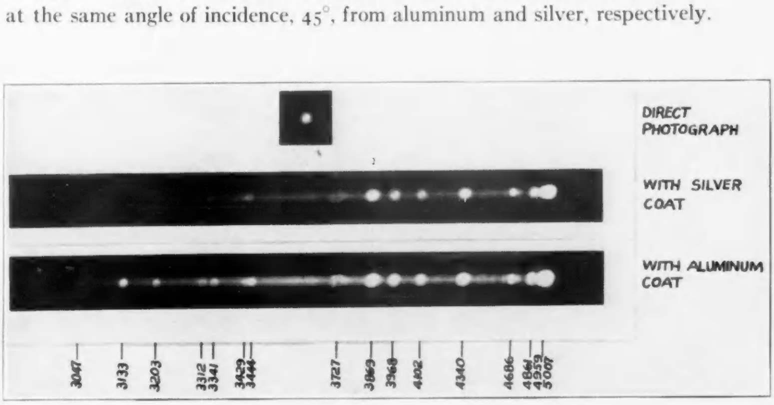
[29] *Pub. of Astr. Soc. of Pac.*, **46**, 32, 1934.

PLATE XVII



SKY SPECTRA

Upper series taken with three reflections from aluminum mirrors at 45° incidence. Each successive spectrum has double the exposure of the preceding one. Second series, three reflections from silver. Third and fourth series, one reflection at the same angle of incidence, 45°, from aluminum and silver, respectively.



SPECTRA OF NGC 7662

Taken with the Crossley reflector of the Lick Observatory by W. H. Wright

culation will agree with the results obtained with the 60-inch and the 100-inch telescopes on Mount Wilson.

Aluminized mirrors scatter less light than silvered mirrors, as the latter have to be burnished and this process leaves many small scratches upon the silver. We may appreciate this difference from the fact that with the fresh aluminum coat it was possible to photograph[30] the companion of Sirius at the Cassegrain focus of the 60-inch telescope on an unbacked plate.

The effects of coating speculum gratings by evaporation is to increase their reflection about 50 per cent. Other effects have been made the subject of a separate paper published in the February 15 issue of the *Physical Review*. Successful results have been obtained by coating replica transmission gratings to change them to reflection gratings.

OXIDE FILMS ON ALUMINUM

The aluminum film is automatically protected from tarnishing by an oxide film (presumably corundum, Al_2O_3, or bauxite, $Al_2O_3 \cdot 2H_2O$) which starts to form as soon as the aluminum comes in contact with the air. This oxide becomes thicker with time for about sixty days, when it is very hard and tough, forming a surface not easily scratched when it is being dusted and cleaned.

The formation of the aluminum oxide film on an exposed metallic aluminum surface has been studied by Vernon,[31] who weighed the sample to a precision of 1/100 mg and plotted the weight increment due to oxidation as a function of time. The period of time required for the oxide film to attain its natural thickness was seven to fourteen days. After this the thickness of the film remained nearly constant at 100 A.

This oxide layer is, of course, too thin to give interference effects. It could be much thicker, however, and still not give interference, because of its transparency and the high reflectivity of the underlying aluminum.[32]

[30] M. H. Humason, *ibid.*, **47**, 83, 1935.

[31] *Trans. Faraday Soc.*, **23**, 113, 1927.

[32] For a discussion of colors of transparent films on metallic surfaces see Robert W. Wood, *Physical Optics*, p. 206, 1934.

It has been observed for a long time that nitric acid does not dissolve an aluminum film. Experiments have shown, however, that treatment with this acid will apparently increase the thickness of the oxide film by about 50 per cent. These conclusions were inferred from measurements of changes in transmission and reflectivity of transparent films, and are based on the assumption that the nitric acid forms aluminum oxide on the surface of the film. The measurements of transmission, T, and reflection, R, given in Table III, are for two plates coated simultaneously with the same thickness of aluminum. One of these, Plate A, was treated with acid, and the other, B, was allowed to oxidize naturally. One sees from the table that the reflection and transmission coefficients, originally $R=0.62$ and $T=0.18$, are changed to $R=0.56$ and $T=0.22$ by one nitric acid bath, and to $R=0.53$ and $T=0.25$ by the second bath.

TABLE III

EFFECT OF OXIDATION OF ALUMINUM ON REFLECTION AND TRANSMISSION

Mirror	Age	Remarks	R	T	$R+T$
A and B	5 min	90 A in thickness	62	18	80
A	10	Washed 45 sec with HNO_3	56	22	78
A	15	Washed 60 sec with HNO_3	53	25	78
A	4 hr		47	29	76
A	18		46	31	77
A	46		46	32	78
B	56	Not washed	53	26	79
A	71		45	31	76
B	71		52	26	78
A	5 days		43	32	75
B	5		51	26	77
A	12		42	33	75
B	12		50	28	78
B	12	Washed 60 sec with HNO_3	50	28	78
A	16		41	35	76
B	16		49	28	77
A	41		37	36	73
B	41		44	30	74
A	61	67 A in thickness	38	35	73
B	61	73 A in thickness	44	30	74

The reflectivity was measured by comparison with an opaque aluminum film, with green light. The reflection of aluminum was taken as 89 per cent. The transmission measurements are uncorrected for glass backing.

The film was originally 90 A thick. If we interpret the changes of T and R as due simply to a decrease in the thickness produced by oxidation of the superficial aluminum, then film A which stabilizes

at $R=0.44$, $T=0.30$, is about 67 A thick. Film B stabilizes at $R=0.38$, $T=0.35$, or at a thickness of approximately 73 A. Assuming the oxide layer to be hydrated alumina ($Al_2O_3 \cdot 2H_2O$), it turns out that the thicker oxide film, A, is 62 A and the thinner one 46 A.[33]

The method of modifying the oxide film by treatment with HNO_3 would seem to have practical application to telescope mirrors in

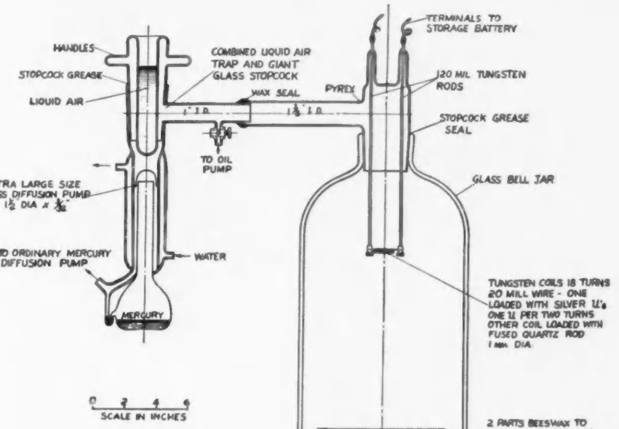


FIG. 6.—First evaporation apparatus constructed at the University of Michigan in 1929.

making the film more resistant to abrasion. The column $R+T$ in Table III, giving the efficiency of transparent aluminum films, is of interest in connection with their use on interferometer mirrors for dividing a beam of light.

EQUIPMENT

The apparatus constructed by the author at the University of Michigan in 1929–1930 is shown in Figure 6. This is, to my knowl-

[33] This thickness was computed, assuming the inverse-square law to hold for the intensity of the evaporated molecular beam. Also zero reflection was assumed for the aluminum beam. It must be remembered that although we have considerable evidence to support these assumptions, they must, nevertheless, be regarded strictly as assumptions. We have, on the other hand, some evidence that the thickness of the aluminized film produced by evaporating a given mass of metal is less when the vacuum is inferior, and at the same time granular aluminum deposits appear on surfaces not exposed to the evaporation sources. Although in the present case the vacuum was above 10^{-4} mm, the numbers associated with these film thicknesses are to be considered as only approximate.

edge, the first apparatus constructed for the general application of the evaporation process to the coating of mirrors. The special large mercury diffusion pump for evacuating the glass bell jar in which the evaporation is carried out was constructed with a large built-in stopcock and a liquid-air trap which permitted the recharging of the evaporation coils and the introduction of new mirrors without destroying the vacuum in the diffusion pumps. Four electrodes allowed several coils of tungsten to be fired. The base plate for the bell jar was of glass. After the assembly of the bell jar on the base plate with the pumping manifold and filament electrodes, the whole was attached to the pumping system by means of a wax joint. In a few minutes, when the wax was cool, the bell jar was evacuated by the vacuum pump through the by-pass. After this, the by-pass was closed, liquid air was introduced into the trap, and the large valve was opened. In a few minutes the vacuum was good enough (i.e., 10^{-4} mm of mercury or better) for the coils of tungsten, containing the metal (or non-metal), to be fired. A minute or two were required for the evaporation of the silver, during which time the bell jar became a beautiful blue as the silver condensed on it, forming a film of increasing opacity.

The next apparatus, developed by Cartwright and Strong, used the absorption of charcoal at liquid-air temperatures to obtain the necessary high vacuum. The evaporation was carried out in a glass bell jar sealed to a steel base by means of a mixture of beeswax and resin (2:1). This apparatus was later equipped with a mercury vapor pump. This is shown in Figure 7, together with the 14-inch cast-aluminum bell jar later constructed for use with it.

In May, 1932, experiments were started preparatory to coating, by evaporation, the 100-inch Hooker mirror of the Mount Wilson Observatory. The first plan considered was to use a small bell jar which would be placed directly on the mirror surface, thus using the mirror as a base plate to seal the vacuum system. This would make it possible, with the help of a baffle, to coat an area of hexagonal shape. By repeating these hexagons side by side it would be possible to cover the entire surface of the mirror. Difficulty was met in finding a material to use for the rim of the bell jar which would give a vacuum-tight seal without injury either to the film already deposited or the mirror face. After considerable experimentation a lacquered hard-

rubber foot about $\frac{1}{8}$ inch wide was found satisfactory. The rubber was mounted in a groove in the rim of the brass bell jar. It was coated with several thoroughly dried layers of colorless brushing lacquer to keep it from sticking to the metallic film already laid down. Preliminary tests were made on a piece of plate glass, indicating that this technique could be made successful.

The plan was abandoned, however, when it became apparent that the construction and evacuation of a tank large enough to coat the whole mirror at once was possible.

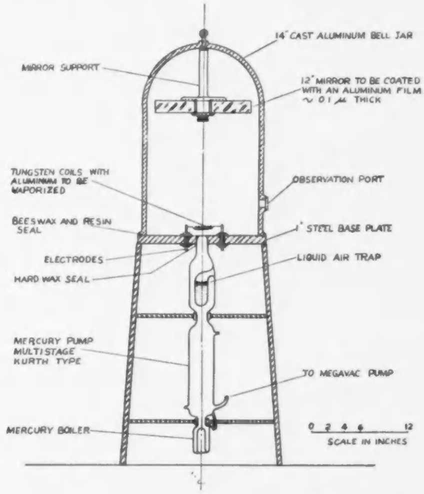


FIG. 7.—Small evaporation apparatus used to coat mirrors to 12-inch diameter as well as for general experimental work.

The next apparatus is shown in Figure 1. This was constructed with the possibility in mind of coating the 36-inch Crossley reflector. It was different from any constructed before in two important respects: it was made of ordinary boiler-plate steel throughout and was arranged to coat the mirrors face upward from coils directly above them. The operation of this tank has proved that the simple method of laying the mirrors on the base plate face upward is entirely successful. It has been used more than a hundred times and we have never had molten aluminum fall upon a mirror.

The 108-inch tank for the 100-inch Hooker mirror is shown in Plate XVI, together with the high-vacuum pumping system constructed for use with it. The tank was made from a steel cylinder 108 inches in diameter, 36 inches high, and $\frac{5}{16}$ inch thick. It was closed at the top with a bumped tank-head $\frac{3}{8}$ inch thick. The cylinder was fitted with two 8-inch ports, one for the electrical connections and the other for the pumping system. At the bottom it was connected to a ring on which a $\frac{1}{2}$-inch tongue was machined. This tongue was seated on a soft rubber gasket, $\frac{1}{2}$ by $\frac{1}{4}$ inch, retained in a groove machined in the base. The base was made from $\frac{5}{16}$-inch plate

reinforced beneath by *I*-beams. A ring was welded to it around the periphery, for strength, and the groove was machined in this to fit the tongue referred to above.

All the joints in this jar were electrically welded with three passes. The jar was X-rayed and stress-annealed before machining. After machining it was tested with 20 pounds per square inch of internal pressure for strength and leaks. The latter were detected with soapy water and repaired by peaning.

Plate XVI shows the 108-inch tank, with pumping system connected, installed in the dome near the 100-inch telescope. The charcoal trap, not used, is shown in the foreground. The four cables connected to the top of the tank lift both tank and pump from the base plate for introducing the mirrors and loading the coils.

HISTORY OF THE APPLICATION OF ALUMINIZING TO ASTRONOMICAL MIRRORS

It is not known to me when the application of the evaporation process to astronomical mirrors was first conceived, but, to my knowledge, the first astronomical mirror to be coated by the process was that of Professor Philip S. Fogg, Registrar of the California Institute of Technology. The author coated his 6-inch mirror, which he had ground and figured himself, with silver and a protecting layer of quartz by the evaporation process on January 14, 1931. Soon after this Cartwright coated several mirrors with silver and quartz for the Hale spectrohelioscope at the Brackett Observatory, Claremont, California.

The first group of mirrors aluminized by the new technique were coated June 19, 1932, by the author. One of these mirrors, which was sent to Albert G. Ingalls of the Scientific American, was not tarnished two years later.

The work at Cornell University on the application of the evaporation process to large astronomical mirrors was started in the summer of 1931, culminating in the coating, with chromium, of the 10-inch concave mirror of the Physics Department, and of the 15-inch mirror of the Lowell Observatory, in July, 1932.

During October, 1932, I coated with aluminum the 12-inch mirror of the Cassegrain telescope of the California Institute. This mirror is now the oldest aluminized telescope mirror, being three years old. The deterioration of its reflectivity has been less than 1 per cent.

R. C. Williams undertook the coating of astronomical mirrors with aluminum in the autumn of 1932, leading to the coating of the 15-inch mirror of the Lowell Observatory in August, 1933. His work ultimately led him to the conclusion that by first making a chromium mirror whose surface is then brightened with aluminum a reflecting film is obtained that is superior to a plain aluminum mirror with respect to hardness and adhesion. He has suggested the name "chroluminum" for this combination.

Hiram W. Edwards has applied the evaporation technique to magnesium alloys of aluminum which are reported to be superior in reflectivity to pure aluminum. He uses the name "panchro" for this coating.

In September, 1933, at the California Institute of Technology, the 18-inch coelostat mirror of Dr. Hale's solar telescope was aluminized by the author. In November, 1934, the mirrors of the 60-foot and 150-foot tower telescopes, as well as several of the Cassegrain, Coudé, and Newtonian auxiliary mirrors of the 60-inch and 100-inch telescopes of the Mount Wilson Observatory, were aluminized. In December, 1933, an aluminum coat was applied by the author to the Crossley reflector of the Lick Observatory. In January, 1934, the 24-inch reflector of the Yerkes Observatory was aluminized.

The largest telescope mirrors that have been coated are the 60-inch (on February 27, 1935) and the 100-inch (on March 1, 1935) of the Mount Wilson Observatory.

I wish to acknowledge the many helpful suggestions I have received from Dr. John A. Anderson, and the assistance of Professor E. Gaviola and of the staff of the Mount Wilson Observatory, during the coating of the 60-inch and 100-inch mirrors; and to thank Director W. H. Wright of the Lick Observatory for supplying the photograph used in Plate XVII. I also wish to acknowledge the valuable advice of Mr. G. W. Sherburne, Mr. Mark Serrurier, and Mr. S. Hart in the design and construction of the 108-inch tank.

Astrophysical Observatory
California Institute of Technology
Pasadena, California
January 17, 1936

PHOTOELECTRIC MAGNITUDES OF THE BRIGHTEST EXTRA-GALACTIC NEBULAE*

ALBERT E. WHITFORD

ABSTRACT

The integrated magnitudes of eleven of the nearest and brightest extra-galactic nebulae have been measured by means of the photoelectric photometer attached to the 10-inch Cooke photographic refractor. The advantage of the photoelectric cell in comparing stars and nebulae is pointed out. The resulting magnitudes average brighter than either the photographic or the visual estimates previously used in computing absolute luminosities of nebulae.

The nearest and brightest extra-galactic nebulae must form the "calibration" for the scale of nebular distances, for only in these systems is it possible to recognize Cepheid variables or other individual stars of known type. From this small sample collection an estimate of the mean absolute luminosity of nebulae in general can be made. Also, some idea of the dispersion about the mean luminosity and variation with type may be obtained. Once equipped with this information, one may infer the distance of a faint nebula from its apparent luminosity. Hubble[1] initiated this scheme and has discussed the problem in detail.

The obvious difficulty has been the comparison of diffuse luminous areas with point sources such as stars. Visual estimates as made by Holetschek[2] or estimates from inspection of photographs such as those by Shapley and Ames[3] are useful as preliminary guides. Hubble[1] has outlined a photographic method of measurement employing extra-focal images which he has used with success. The photoelectric cell seems well adapted to a measurement of this sort because it integrates all the light that comes into it and gives an electric current proportional to the total brightness.[4] It was first used to meas-

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 543.

[1] *Mt. Wilson Contr.*, No. 427; *Ap. J.*, **74**, 43, 1931.

[2] *Ann. d. Wiener Sternwarte*, 1907; corrections by Hopmann, *A.N.*, **214**, 425, 1921.

[3] *Harvard Ann.*, **88**, No. 2, 1932.

[4] This statement is not only theoretically true, at least to a first approximation, but can also be justified by experience. The galvanometer deflections due to the sky when

ure the light of nebulae at Mount Wilson by Dr. Joel Stebbins in 1930, and has been so used by him, often with the assistance of the writer, at various times since.[5] The measurement of the nearest nebulae was undertaken by the writer at the suggestion of Messrs. Hubble, Baade, and Stebbins. It is just for these cases where the total brightness is of the most importance that extra-focal photographic methods become difficult or break down because of the great angular extent of the nebulae, and the photoelectric cell, therefore, appears to be the most feasible device to use.

Observations have been made on eleven nebulae. These include ten of the fourteen nebulae north of $-30°$ declination and brighter than the tenth magnitude in Shapley and Ames's compilation. To these was added NGC 5194 (M51), because individual stars have been resolved in this nebula and it was believed to be brighter than magnitude 10.1 as listed.

A short-focus telescope was required in order to include the necessary area in the field of view of the photocell. Consequently the photometer as used on the large reflectors was adapted to the 10-inch Cooke photographic telescope. The arrangements did not differ in principle from those described for the Washburn Observatory.[6] The galvanometer deflections were recorded photographically on bromide paper by means of a motor-driven drum, which made it possible for the observations to be made entirely by one person. The largest focal-plane diaphragm that could be used was 10 mm in diameter, corresponding to a field of 30 minutes of arc.

The precision of the magnitudes is not so great as is usually obtained with the photoelectric photometer. In the first place, owing to the limited time, it was not feasible to measure each nebula for a series of increasing diameters. Instead, usually a diaphragm was selected large enough to include the diameter published by Shapley

no stars can be seen in the field is closely proportional to the measured area of the diaphragm in the focal plane. The proportionality of current to light intensity has been tested on well-calibrated sequences of stellar magnitudes, and also with a rotating sector, and found to be satisfactory over the entire usable range.

[5] An account of this investigation is to be published in the *Astrophysical Journal*. The writer is indebted to Dr. Stebbins for permission to publish the present paper in advance of his own work in the same field.

[6] *Ap. J.*, **76**, 213, 1932.

and Ames. This meant that the correction for light from the sky background was as large as 70 or 80 per cent of the total deflection for some of the larger nebulae. The relative brightness of the sky and the fact that many of the magnitudes are near the usable limit of the amplifier with a 10-inch telescope explain why discrepancies larger than usual may be expected. Another factor was the difficulty of dodging foreground stars. With the large reflectors it is easier to do this, owing to the well-known thinning-out of stars at the fainter magnitudes. For example, suppose stars are to be excluded down to the twelfth magnitude with the 10-inch and to the seventeenth magnitude with the 100-inch telescope. Seares and van Rhijn[7] find at the galactic pole 9.9 stars per square degree brighter than the twelfth photographic magnitude, and 288 stars per square degree brighter than the seventeenth photographic magnitude. If the slight difference in focal ratio between the two telescopes be neglected, there will be, on the average, $990/288 = 3.4$ times as many stars visible in a given area in the focal plane of the 10-inch as in the same area in the focal plane of the 100-inch telescope. When a star had to be included in the field of a nebula, a note was made of the fact and an attempt to correct for it was made. A comparison field free from stars was often difficult to find. Internal evidence indicates that varying success in choosing proper comparison areas on different nights accounts for a large part of the discordances.

Messier 31, the great nebula in Andromeda, is a special case because of its great angular area on the sky. The central region included in a circle half a degree in diameter could be measured with the 10-inch telescope, and this was done. In order to get the light from the whole nebula into the photocell, it was necessary to use a lens of shorter focus. A Tessar lens of $3\frac{1}{2}$ inches aperture and 16 inches focus, with suitable adapters for holding the amplifier, was bolted to the 10-inch mounting. Diaphragms up to 20 mm in diameter could be used in this arrangement, corresponding to a field of 170 minutes of arc. With several square degrees in the field, at a fairly low galactic latitude, it was apparent that corrections for the field stars unavoidably included would have to be made. Two comparison areas free from very bright stars were selected near the

[7] *Mt. Wilson Contr.*, No. 301; *Ap. J.*, **62**, 320, 1925.

nebula. Then the total light of all the *Bonner Durchmusterung* stars in the field of the nebula and in the two comparison areas was computed. As far as possible, the *Draper Catalogue* was used for the magnitudes, reductions to photoelectric magnitude being applied according to the spectral class. For stars too faint to be included in the *Draper Catalogue*, which is complete only to the eighth visual magnitude, the BD magnitudes were used, with Pickering's corrections.[8] An average color index corresponding to a G0 star was assumed. The average color index for 60 stars in the *Draper Catalogue* fainter than visual magnitude 8.0, all in the region of the nebula, corresponded to an F9 star. Since these stars are individually unimportant in determining the total light, an approximation of this nature is satisfactory. The telescope was set on the comparison areas by means of the circles, with the nebula as a reference point. It is evident from the results that it would have been better to use a particular star as the center of each area.

The diaphragm finally used in computing the magnitude of M 31 was 15 mm in diameter, corresponding to 128 minutes of arc. A diameter larger than this necessarily included several comparatively bright stars, among them one brighter than the whole nebula, which would have made the corrections rather large and uncertain. Within the field of view, 128 minutes in diameter, on one particular night the light of the nebula was 19 per cent of the total light; the light of the stars listed in the *Bonner Durchmusterung* was 9 per cent; and the light of the sky, including fainter stars, accounted for the remainder. The correction for difference in calculated starlight between the area around the nebula and the comparison area was at most 11 per cent of the light of the nebula.

The measurements indicate that a circular area 30 minutes in diameter, including the very bright nuclear region, gives out only 40 per cent as much light as those parts of the nebula included in a circle 128 minutes in diameter. Because this result is a little surprising, a rough computation of the total light from the relatively faint outer regions was made. The nebula was divided into elliptical zones whose semimajor axes differed by 15 minutes. The ratio of the major to the minor axis was taken as 4:1. The area of each zone was

[8] *Harvard Ann.*, **72**, 191, 1913.

computed, portions within the central 30-minute circle being excluded. The average brightness of each zone was evaluated in two ways. The first method involved the use of a photograph of the nebula with the 10-inch telescope, made by the writer for another purpose, but provided with calibration for sensitometry. The zero point depended on an estimated sky brightness for the night in question of 22.0 mag. per square second. The second method depended on photoelectric measures along a circle of constant declination through the nucleus, made by Dr. Stebbins and the writer with the 100-inch telescope in the summer of 1934. Either method gives a result for the total light of the outer regions of the right order of magnitude. The explanation of this relatively large total lies, of course, in the much greater area of the outlying zones.

The nebulae were compared with seventeen stars, all between the second and the fifth magnitudes. Because these stars are bright, they are easy to find and measure accurately, with no sky correction. The scale of magnitudes can then be carried to fainter objects by varying the shunt on the galvanometer or by using a higher resistance in series with the photocell. It is of course desirable that the photoelectric magnitudes be converted to the photographic system. The potassium hydride cells used in this work have a maximum response at 4500 A[9] and differ only slightly in their color response from the photographic plate.

In order to connect the magnitudes with the international system, certain key stars were later recompared among themselves; and two of them, β Cassiopeiae and β Ursae Majoris, were compared with six stars of the North Polar Sequence. This was done in Madison, Wisconsin. Of course this involved a different telescope and photoelectric cell, but the color differences were found to be negligible. The adopted photoelectric magnitudes of the polar stars and of six of the comparison stars for nebulae are given in Table I. The relation between the *IPg* system and *Pe* magnitudes is

$$IPg = Pe + 0.18\, C_e\ ,$$

where C_e is the color index found by Seares by the method of exposure ratios.[10] This equation was derived from an intercomparison

[9] *Pub. Washburn Obs.*, **15**, Part 5, 1934.

[10] *Trans. I.A.U.*, **1**, 71, 1922.

of eight polar stars made with the 60-inch telescope, which will be published in another paper. For the four stars common to the two lists, the 60-inch results were taken as standards because of the higher precision of the measures and the better atmospheric conditions at Mount Wilson. Any other selection of polar stars would not change

TABLE I

COMPARISON STARS

Star	HR	Spectrum	*Pe* Mag.	Calculated *Pg* Mag.	*I Pg* Mag.
β Cass	21	F5	2.56	....	....
β Arietis	553	A5	2.76	....	....
α Arietis	617	K2	2.98	....	....
β Urs Maj	4295	A0	2.32	....	....
α Urs Maj	4301	K0	2.66	....	....
ι Pegasi	8430	F5	4.13	....	....
NPS 3	8938	F0	5.66	5.69	5.78
4	6811	A3	5.86	5.90	5.91
1r	2609	Ma	6.39	6.67	6.69
5	286	A2	6.45	6.45	6.46
6	....	A0	7.15	7.16	7.12
2r	7394	Mb	7.67	7.95	7.93

the zero point of the magnitude scale by more than 0.01 or 0.02 mag. Since the accuracy of the nebular comparisons is not much better than 0.1 mag., this uncertainty is negligible.

Measures of some sixty nebulae with the photoelectric cell gave an average color corresponding to a G4 star. The revised[11] value of the international color index for a giant star of this type is very nearly +0.7 mag., so that for the average nebula the foregoing equation becomes

$$I\,Pg = Pe + 0.13 \ .$$

RESULTS

The results are summarized in Table II. The diameter and classification and also the Harvard photographic magnitudes are taken from Shapley and Ames's publication. The observed photoelectric magnitudes are based on the magnitudes of the six comparison stars listed in Table I, which have been standardized against the North Polar Sequence. The errors given in the seventh column are the

[11] Seares, private communication.

TABLE II

MAGNITUDES OF NEBULAE

NGC	Messier	Type	Diameter	No. of Obs.	Diaphragm	Obs. *Pe* Mag.	Adopted *Pg* Mag.	Harvard *Pg* Mag.	Hopmann *Pg* Mag.
221	M32	E	2.′6×2.′1	2	7.′5	9.24±.13	9.4	9.5	9.9
224	M31	Sb	160×40	2 3 4	24 30 128	5.45±.03 5.35±.04 4.31±.09	4.5	5±	6.1
253	......	Sc	22×6	2 2	12 24	8.18±.18 7.94±.08	8.1	7.0	....
598	M33	Sc	60×40	2 2 1	24 30 54	6.95±.13 6.76±.16 6.91	6.9	7.8	8.1
3031	M81	Sb	16×10	3 3	12 24	8.04±.05 7.56±.23	8.0	8.9	9.4
3034	M82	Irr	7×1.5	2 2	7.5 12	9.14±.18 8.82±.02	9.1	9.4	10.1
4594	......	Sa	7×1.5	1	7.5	9.11	9.2	8.1	10.3
4736	M94	Sb	5×3.5	2	7.5	8.82±.08	9.0	9.0	9.5
4826	M64	Sb	8×4	2	12	9.30±.06	9.4	8.0	10.3
5194 5195	M51	Sc	12×6	3	12	8.83±.15	9.0	9.7	8.5
5457	M101	Sc	22×22	4	24	8.68±.12	8.8	9.0	....

NOTES TO TABLE II

NGC 221 Half-weight assigned to observations on one night because only one deflection for sky area was obtained.

NGC 224 Result for 128′ diaphragm from observations on two nights with 3½-inch lens, two comparison areas each night. Corrected for field stars as explained in text. *Pg* mag. = 4.44; adopted 4.5.

NGC 253 Smaller diaphragm excludes field stars. Larger diaphragm includes two stars. Brighter star measured photoelectrically, found *Pe* mag. = 10.3; fainter one imbedded in nebulosity, but estimated from photograph to be at least 0.5 mag. fainter. Assumed 0.6 mag. fainter. Obs. *Pe* mag. already corrected.

NGC 598 Result for 54′ diaphragm from observations with 3½-inch lens, corrected for field stars as explained in text.

NGC 3031 Stars excluded by smaller diaphragm. Obs. *Pe* mag. for larger diaphragm includes light from two stars; correction estimated from BD for adopted *Pg* mag.

NGC 3034 Obs. *Pe* mag. for both diaphragms includes light of star, correction estimated for adopted *Pg* mag.

NGC 5194, NGC 5195 Both nuclei included in field. Observation on one night given half-weight because of very large extinction correction. If, as Shapley estimates, the two nebulae differ by 1 mag., then NGC 5194 = 9.4 mag. and NGC 5195 = 10.4 mag. Magnitudes given in table are for combined light in each case.

NGC 5457 Observations on two nights given half-weight because of very large extinction corrections.

average deviations from the means. The adopted photographic magnitude was in each case obtained by applying a color correction of +0.13 mag. to the weighted mean of the photoelectric magnitudes for the whole diameter of the nebula, corrected for field stars when necessary. The figures in the last column were derived from Hop-

TABLE III

ABSOLUTE PHOTOGRAPHIC MAGNITUDES OF NEBULAE

Nebula	Modulus $m-M$	m	M
M31	22.0	4.5	−17.5
M32	22.0	9.4	12.6
M33	21.9	6.9	15.0
M81	24.3	8.0	16.3
M101	23.0	8.8	−14.2

mann's visual magnitudes by applying a color correction of +1.1 mag., as used by Hubble, to convert them to the photographic system.

The systematic difference between the Harvard magnitudes and the magnitudes here presented is less than 0.1 mag., with an average deviation from the mean of ±0.65 mag. However, with three exceptions, including NGC 4594 in which no individual stars have been resolved, the present magnitudes run slightly brighter than the Harvard estimates. Hopmann's results, based on Holetschek's visual estimates, show a systematic difference of 0.85 mag. relative to the present results, with the present results brighter. The average deviation from the mean is ±0.45 mag.

The data presented in this paper are not sufficient to warrant a new discussion of the mean absolute magnitude of a nebula. Such a discussion must await revised magnitudes of all the nebulae in which

individual stars have been resolved. It is of interest, however, to compute the absolute magnitudes of the five nebulae whose moduli are known[1] from the variables resolved in them. These results are shown in Table III. Because there is reason to believe that these nebulae may not be a fair sample, it would be unsafe to infer too much from these figures. Nevertheless, it may be remarked that the results of this work seem to point to a value for the mean absolute photographic magnitude of a nebula somewhat brighter than the mean adopted by Hubble[1] in his original paper on the basis of the nebular magnitudes available to him at that time.

Carnegie Institution of Washington
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THE PROVISIONAL ELEMENTS OF 16 SPECTROSCOPIC BINARIES*

WILLIAM H. CHRISTIE

ABSTRACT

Provisional elements, derived graphically from various published lists of radial velocities, are given for 16 late-type spectroscopic binaries.

The provisional orbits for the accompanying list of spectroscopic binaries have been derived from various published radial velocities of these stars, supplemented, in some cases, by Mount Wilson observations.

After the periods had been obtained in the usual manner, the elements in Table I were derived by comparing the plotted observations with a series of standard velocity-curves. The first eleven columns need no explanation; columns 12, 13, 14, and 15 contain the numbers of observations made at the Cape, Lick, Mount Wilson, and Victoria observatories, respectively; column 16 gives references to the sources of the material; and column 17, the number of the accompanying figure which shows the velocity-curve and the observations.

In order to save space in printing, the original observations are not repeated here; the numeral placed near the plotted observation refers, however, to the number of the observation in the original list; thus, a 7 near a filled circle indicates that the observation is the seventh in the Lick Observatory list for this star. Similarly, 3–8 beside a vertically half-filled circle denotes that the point is a normal for observations 3–8 in the Victoria list.

NOTES

Boss 500.—Two observations on this curve are discordant, Nos. 23 and 25. A great deal of effort has been made to find a shorter period which will satisfy the observations, but without success. It seems likely that the elements are not seriously in error, despite the two large residuals. Four kilometers per second have been added

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 544.

TABLE I

ELEMENTS OF SIXTEEN SPECTROSCOPIC BINARIES

STAR	R.A. 1900	Dec. 1900	Mag.	Type	P	e	ω	γ	K	T	OBSERVATIONS				REFERENCES	FIG.
											C	L	MW	V		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
					days			km/sec	km/sec	JD						
Boss 500	$2^h 07^m.0$	+50° 36′	5.4	K0	1650	0.75	270°	+27.3	13.5	2,425,200	.....	6	11	28	a, d	2
τ Persei	2 47.2	+52 21	4.1	G5–A5	1520	0.77	240	+ 2.2	20.0	2,415,695	.....	21	.....	2	a, d	3
16 Aurigae	5 11.6	+33 17	4.8	K0	434.8	0.1	40	−27.5	14.8	2,418,690	.....	5	18	2	a, d	16
η Geminorum	6 08.8	+22 32	Var	M3	1875	0.4	280	+18.2	5.3	2,421,260	3	11	4	2	a, b, d	4
a Puppis	7 48.8	−40 19	3.8	G5	2660	0.4	190	+24.0	11.8	2,417,165	12	7	.....	.....	a, b	5
h² Puppis	8 10.5	−40 02	4.4	K0	930	0.4	140	+13.5	10.8	2,418,060	18	7	.....	.....	a, b	6
ϕ Hydrae	10 33.7	−16 22	5.1	K0	1200	0.1	270	+16.2	4.0	2,420,760	.....	10	.....	.....	a	1
Boss 3195	12 11.5	+33 37	5.1	K0	1300	0.3	300	−41.5	6.8	2,421,750	.....	8	3	.....	a	7
δ Muscae	12 55.4	−71 01	3.6	K2	847	0.4	0	+36.5	7.8	2,421,790	11	12	.....	.....	a, b	8
υ^2 Centauri	13 55.5	−45 07	4.4	F5	1025	0.4	140	− 0.5	8.2	2,423,960	7	5	.....	.....	a, b	14
HR 5752	15 25.6	+47 35	6.0	A3	105.8	0.1	30	−17.0	11.2	2,424,346	.....	.....	.....	.....	See text	13
Boss 4129	16 08.1	+36 41	5.7	K4	2150	0.6	340	−30.6	16.0	2,424,290	.....	.....	11	24	c, d	9
Boss 4351	17 02.1	+48 56	6.3	K0	786	0.3	0	+12.8	10.0	2,426,275	.....	.....	109	6	c	15
δ Sagittae	19 42.9	+18 17	3.8	Ma–A0	3988	0.75	270	+ 1.4	13.0	2,422,500	.....	9	5	18	a, d	10
o^2 Cygni	20 10.5	+46 26	4.0	K0–B8	4000	0.3	200	Var?	20.0	2,414,400	.....	18	3	2	a, d	11
ν Octantis	21 30.4	−77 50	3.7	K0	1020	0.4	80	+34.0	8.0	2,418,525	10	11	.....	.....	a, b	12

[a] *Lick Obs. Pub.*, **16**, 1928.
[b] *Cape Ann.*, **10**, Part 8, 1928.
[c] *Pub. Dom. Ap. Obs.*, **2**, Part 1, 1924.
[d] *Ibid.*, **6**, Part 10, 1931.

to the Victoria observations in order to make them fit the curve. This correction seems to be necessary in several cases, probably because of a systematic difference between spectrocomparator and micrometer measures. The star is under observation here.

τ Persei.[1]—It seems strange that these elements have not previously been published, since the 21 Lick observations define the orbit very well.

16 Aurigae.—This orbit has been under investigation here since 1933, and the elements appear to be quite satisfactory. The period is well defined by the early Lick observations.

η Geminorum.—The first Cape observation has been omitted; the residuals are otherwise quite satisfactory.

a Puppis.—Despite the lack of observations following periastron, the elements are probably reliable.

h^2 Puppis.—Further observations of this star should be made, as the derived elements are not too well defined.

φ Hydrae.—These elements appear to be satisfactory, but more observations are needed.

Boss 3195.—Eight observations are, as a rule, hardly sufficient to assure a good orbit. The distribution of these observations is such, however, that the publication of the orbit seems justified.

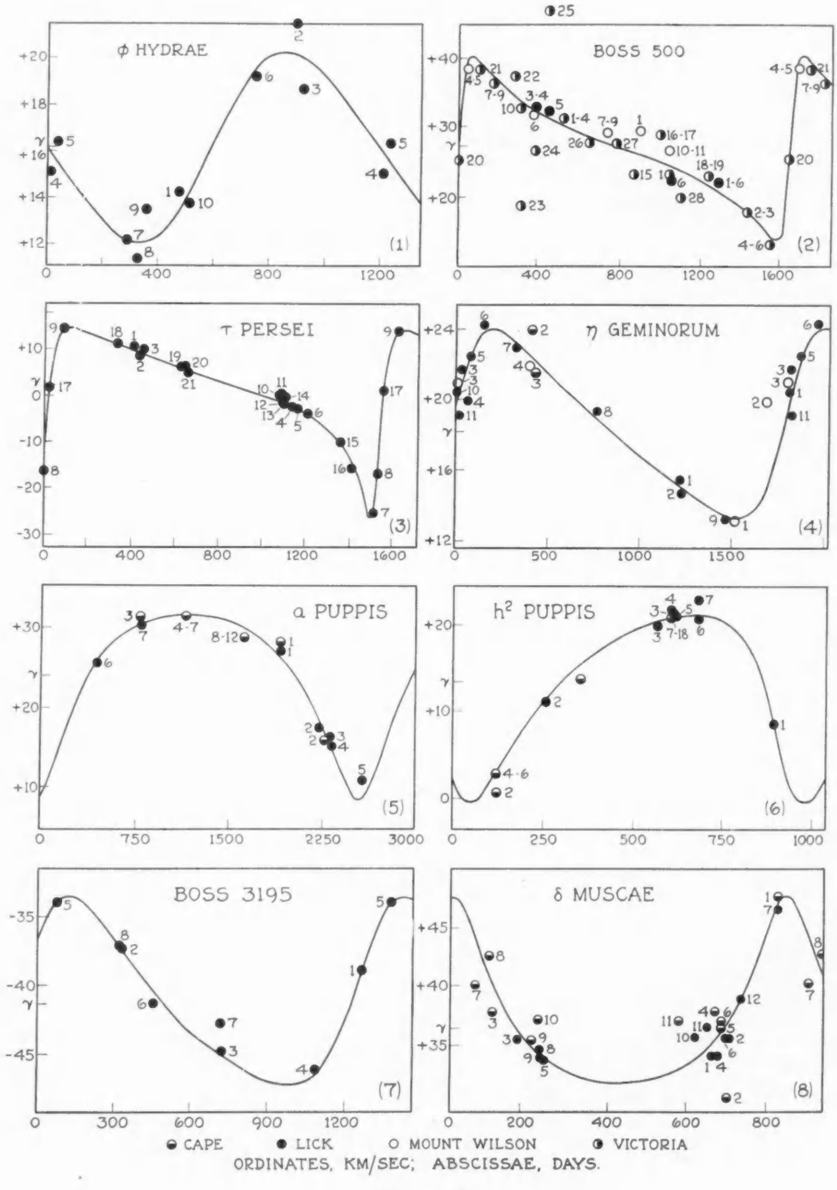
δ Muscae.—The run of the Lick and Cape observations from 1919 to 1922 appears to define the period. Observations are needed at apastron in order to improve the elements. The provisional elements are probably not far wrong, however.

v^2 Centauri.—Although the observations are rather few, the orbit appears to be well defined.

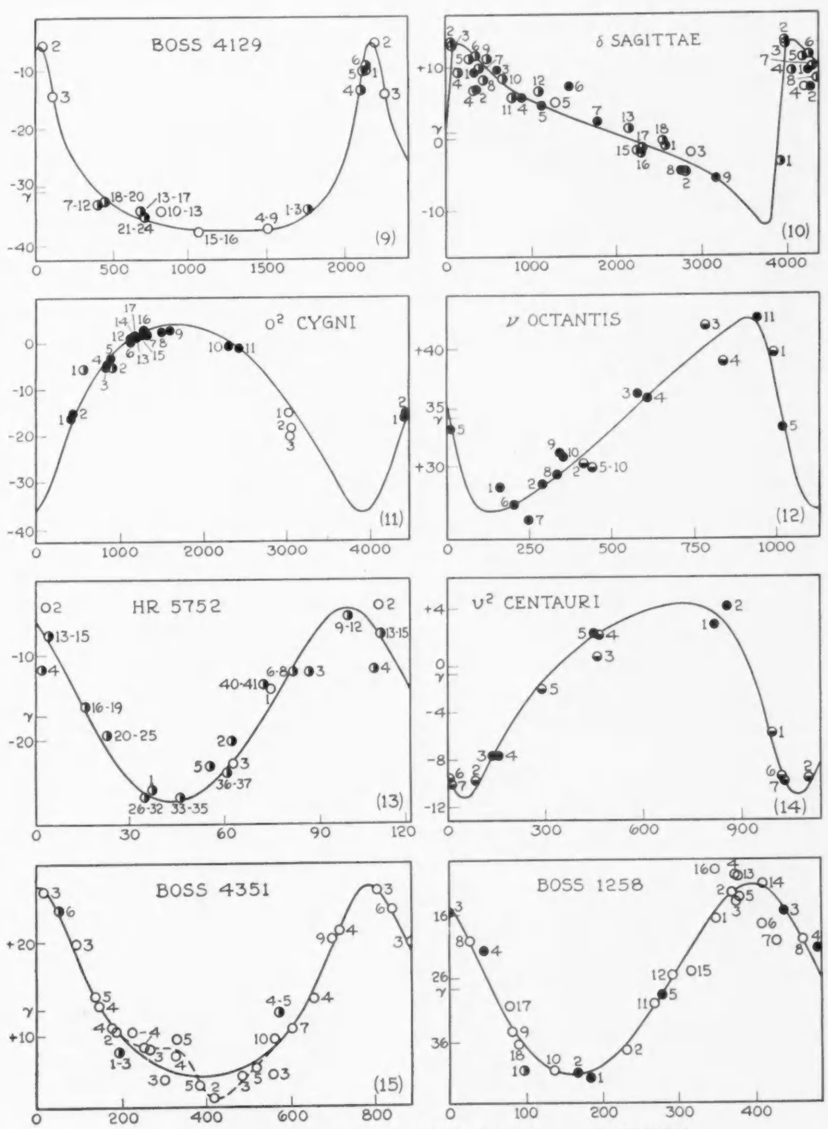
HR 5752.—An orbit for this star was published by the writer in 1926.[2] The period then used turns out to be erroneous. This error was suspected by Hertzsprung, who notified the writer. Spectrograms have recently been taken here on several nights at widely differing hour angles, with the result that the short period has been abandoned in favor of the longer. These new elements are apparently quite satisfactory.

[1] Since this paper was written, the orbit of this star has been published by Colacevich, *Pub. A.S.P.*, **48**, 32, 1936.

[2] *Pub. Dom. Ap. Obs.*, **3**, 310, 1926.


φ HYDRAE
BOSS 500
τ PERSEI
η GEMINORUM
α PUPPIS
h² PUPPIS
BOSS 3195
δ MUSCAE
CAPE
LICK
MOUNT WILSON
VICTORIA
ORDINATES, KM/SEC; ABSCISSAE, DAYS.

FIGS. 1–8



CAPE ● LICK ○ MOUNT WILSON VICTORIA

ORDINATES, KM/SEC; ABSCISSAE, DAYS.

FIGS. 9–16

Boss 4129.—Provisional elements for this star have already been published by the writer.[3] These new elements differ slightly from the former and are probably more accurate.

Boss 4351.—The investigation of this orbit has been under way here since 1929. There is an apparent departure from a normal velocity-curve at apastron. Other widely different velocity-curves also represent the observational data, but the one here published seems the most probable. Observations will be continued. The individual velocities are too numerous to publish here.

δ *Sagittae.*—The period of this star appears to be fairly well defined by the observations. It is necessary, however, to add 4 km/sec to the Victoria observations in order to make them agree with the Lick. This orbit is under further investigation here.

o^2 *Cygni.*—The period is apparently well defined by the Lick observations, and there is some evidence of a change in γ, amounting to about $+0.66$ km/sec per 1000 days, for which the observations have been corrected. The star is under observation here. The Cepheid-type spectrum of the star throws some doubt, however, on these elements.

ν *Octantis.*—Although further observations are desirable, the elements given here appear to be reliable.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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[3] *Pub. A.S.P.*, **46**, 238, 1934.

A QUANTITATIVE INVESTIGATION OF SPECTRAL LINE INTENSITIES IN O- AND B-TYPE STARS*

PAUL RUDNICK

ABSTRACT

Measured total absorptions are given for the stronger spectral lines in seventy stars of Harvard types O and B. The intensities were measured on one-prism spectrograms with a photoelectric microphotometer, using a square diaphragm large enough to include the entire line; in this manner some lines could be measured that were too faint and broad to be seen.

Behavior of H lines.—The behavior of hydrogen intensities with spectral type is shown, first for all of the stars measured, and then for Struve's giant and dwarf groups. The marked enhancement of hydrogen in the dwarfs is confirmed. It is shown that there is no systematic difference in the hydrogen intensities between diffuse-line and sharp-line stars.

Behavior of He I *lines.*—The lines of the triplet system of *He* I are shown to fade out more slowly than the singlet lines on the high-temperature side of maximum; no appreciable difference is found between the systems on the low-temperature side of maximum. Intensities of all *He* I lines, when plotted against spectral type, show a much more definite maximum in dwarfs than in giants. The suspected shift of maximum from B2 in dwarfs to B1 in giants is confirmed. Differences are shown to exist between giants and dwarfs in the behavior of the singlet-to-triplet ratio. The enhancement of *He* I lines in dwarfs in types earlier than B3 is confirmed. At type B2, the strongest lines measured in both singlet and triplet systems are about 50 per cent strengthened in dwarfs as compared with giants. At B2, the triplet λ 4472 is about 50 per cent stronger than the singlet λ 4388. It is shown that the differences between singlets and triplets in giants and dwarfs cannot be due entirely to gradient effect.

The position of maximum intensity of the helium lines is found to be B2 for stars with sharp lines; for wide-line stars it is shifted to B4.

Behavior of lines of other elements.—The behavior with spectral type is given for lines of the following elements: *Si* II, *Si* III, *Si* IV, *He* II, *N* II, *N* III, *C* II, and *Mg* II. The strengthening of λ 4481 of *Mg* II in giants of type B5 and later is found to be about 40 per cent. *Si* II, *Si* III, *Si* IV, *N* II, and *N* III are all found to have about twice the maximum intensity in giants that they attain in dwarfs. Almost all of these elements are found to show no appreciable difference in behavior between wide-line and sharp-line stars.

I. INTRODUCTION AND OBSERVATIONAL MATERIAL

In order to discuss on a uniform basis the intensities of spectral lines of important elements, with various amounts of broadening or with appreciable separation in the spectrum, it seemed worth while

* [This article was received by the editors a few days later than the papers by E. G. Williams printed in the May issue of this *Journal*. Although there is a certain amount of duplication in these two independent investigations, the results are believed to be of sufficient importance to justify the publication of both. No attempt has been made to co-ordinate the results, but the agreement in all the major conclusions is very satisfactory.—THE EDITORS.]

to secure a consistent set of measured total absorptions of all lines strong enough to be well measurable, for a large number of early-type stars. Some work of this nature has already been done by E. G. Williams,[1] who made photometric determinations of line intensities in groups of giant and dwarf O- and B-type stars. Struve[2, 3] has made studies of larger groups of the same types, based mainly on eye estimates of the relative strengths of lines, with special reference to the behavior of *He* I and *O* II. Miss Payne[4] has investigated the behavior, with changing spectral type and with luminosity, of the spectra of numerous atoms; she has worked both with measured line profiles and with estimated data. Elvey has studied profiles of lines of *H*,[5] *He* I,[6] *Si* III,[7] and *Mg* II[8] in the early-type stars.

In the present work, photometric measures of line intensities were made, by the method described below, for the stronger lines of seventy stars of Harvard spectral types Oe to B9. On the basis of the data obtained from these measures, discussions are given of (1) anomalies in the behavior of neutral helium and (2) certain differences in behavior between spectra having sharp and diffuse lines.

The observational material includes eighty-two spectrograms of seventy stars. All but two of the plates were taken with the one-prism dispersion of the Bruce spectrograph (26 A/mm at $H\gamma$); the other two were taken with three prisms (8 A/mm at $H\gamma$). Table I gives for each star the following data, obtained almost entirely from Schlesinger's *Catalogue of Bright Stars:* BS number; name of star; equatorial co-ordinates for 1900.0; apparent magnitude; Harvard spectral type; Victoria spectral type;[9] classification (to be discussed hereinafter) as giant or dwarf and as to line-width class; number of spectra measured.

[1] *Pub. A.S.P.*, **46**, 292, 1934.

[2] *Ap. J.*, **74**, 225, 1931.

[3] *Ibid.*, **78**, 73, 1933.

[4] *The Stars of High Luminosity*, especially chap. xv, 1930; *M.N.*, **92**, 368, 1932. See also *Harvard Circ.*, Nos. 252 and 256, 1924; *Nature*, **113**, 783, 1924; *Harvard Circ.*, No. 365, 1931.

[5] *Ap. J.*, **71**, 191, 1930.

[6] *Ibid.*, **70**, 141, 1929.

[7] *Ibid.*, **78**, 219, 1933.

[8] *Ibid.*, **71**, 221, 1930.

[9] Taken from *Pub. Dom. Ap. Obs.*, **5**, No. 2, 1931, with the exception of 67 Oph, which is there given as B8s; in *ibid.*, No. 1, however, this star is called B3s. I have therefore regarded its Victoria type as B5.

TABLE I

OBSERVATIONAL DATA

BS No.	Star	1900		Mag.	Spectrum		Luminosity Class	Line-Width Class	No. of Plates
		α	δ		Harv.	Vict.			
1228	ξ Per	$3^h52^m.5$	$+35°30'$	4.05	Oe5	O7nk	*d*	II	1
1879	λ Ori	5 29.6	+ 9 52	3.66	Oe5	O8sk	g	I	1
1895	θ^1 Ori	5 30.4	− 5 27	5.36	Oe5	O7k	*d*	I	1
1896	Bond 640	5 30.4	− 5 27	6.85	Oe5	B0	*d*	II	1
1899	ι Ori	5 30.5	− 5 59	2.87	Oe5	O8sk	*d*	II	2
2456	S Mon	6 35.5	+ 9 59	4.68	Oe5	O7sk	*g*	I	2
2781	29 CMa	7 14.5	−24 23	4.90	Oe	O9sfk	g	I	2
2782	τ CMa	7 14.6	−24 46	4.40	Oe5	O9sk	d	II	1
8428	19 Cep	22 2.1	+61 48	5.17	Oe5	O9k	g	I	2
8622	10 Lac	22 34.8	+38 32	4.91	Oe5	O9sk	d	I	1
130	κ Cas	0 27.3	+62 23	4.24	B0	B0k	g	I	1
1542	α Cam	4 44.1	+66 10	4.38	B0	O9sek	g	II	1
1852	δ Ori	5 26.9	− 0 22	2.48	B0	B0k	d	II	2
1876	ϕ^1 Ori	5 29.3	+ 9 25	4.53	B0	B0ssk	d	I	3
1903	ϵ Ori	5 31.1	− 1 16	1.75	B0	B0k	*g*	I	1
1948	ζ Ori	5 35.7	− 2 0	2.05	B0	B0nk	*d*	II	1
2004	κ Ori	5 43.0	− 9 42	2.20	B0	B0k	g	I	1
4133	ρ Leo	10 27.5	+ 9 49	3.85	B0p	B0sk	g	II	1
6165	τ Sco	16 29.6	−28 1	2.91	B0	B1s	d	I	1
1203	ζ Per	3 47.8	+31 35	2.91	B1	B1s	g	II	1
1220	ϵ Per	3 51.1	+39 43	2.96	B1	B2	*d*	II	1
2294	β CMa	6 18.3	−17 54	1.99	B1	B1ss	g	I	2
2618	ϵ CMa	6 54.7	−28 50	1.63	B1	B1s	g	II	1
5984	β Sco	15 59.6	−19 32	2.90	B1	B2k	d	II	1
39	γ Peg	0 8.1	+14 38	2.87	B2	B2ss	d	I	2
779	δ Cet	2 34.4	− 0 6	4.04	B2	B2s	d	I	3
1790	γ Ori	5 19.8	+ 6 16	1.70	B2	B2s	d	II	1
2135	χ^2 Ori	5 58.0	+20 8	4.71	B2p	B2ssk	g	I	1
8279	9 Cep	21 35.2	+61 38	4.87	B2p	B2sk	g	II	1
153	ζ Cas	0 31.4	+53 21	3.72	B3	B2sk	d	I	1
154	π And	0 31.5	+33 10	4.44	B3	B3	d	I	1
542	ϵ Cas	1 47.2	+63 11	3.44	B3	B5s	d	I	1
1239	λ Tau	3 55.1	+12 12	3.3–4.2	B3	B3	*d*	II	1
1641	η Aur	4 59.5	+41 6	3.28	B3	B3	d	II	1
4590	31 Crv	11 55.7	−19 6	5.28	B3	B4n	*d*	II	1
5191	η UMa	13 43.6	+49 49	1.91	B3	B3n	*d*	II	1
6431	u Her	17 13.6	+33 12	4.8–5.3	B3	B3	*d*	II	1
6453	θ Oph	17 15.9	−24 54	3.37	B3	B2	d	I	1
6588	ι Her	17 36.6	+46 4	3.79	B3	B3s	d	I	2
6787	102 Her	18 4.5	+20 48	4.32	B3	B2sk	d	II	1
7298	η Lyr	19 10.4	+38 58	4.46	B3	B5s	d	I	1
7474	σ Aql.	19 34.3	+ 5 10	5.17	B3	B3	*d*	II	1
8926	1H. Cas	23 25.4	+58 0	4.89	B3	B3k	*d*	II	1

TABLE I—*Continued*

BS No.	Star	1900 α	1900 δ	Mag.	Spectrum Harv.	Spectrum Vict.	Luminosity Class	Line-Width Class	No. of Plates
280	α Scl	$0^h53^m.8$	$-29°54'$	4.39	B5	B5	*d*	I	1
1149	20 Tau	3 39.9	+24 4	4.02	B5	B9s	d	I	1
2343	ν Gem	6 23.0	+20 17	4.06	B5	B5ne	*d*	*I*	1
2657	γ CMa	6 59.2	−15 29	4.07	B5	B8	*d*	I	1
2745	27 CMa	7 10.2	−26 11	4.66	B5p	B5	*d*	II	1
6092	τ Her	16 16.7	+46 33	3.91	B5	B7s	d	I	1
6396	ζ Dra	17 8.5	+65 50	3.22	B5	B8s	d	I	1
6714	67 Oph	17 55.6	+ 2 56	3.92	B5p	*B5*?	g	I	1
7852	ϵ Del	20 28.4	+10 58	3.98	B5	B7	d	I	1
7963	λ Cyg	20 43.5	+36 7	4.47	B5	B6e	*d*	II	1
8523	2 Lac	22 16.9	+46 2	4.66	B5	B5	d	I	1
936	β Per	3 1.6	+40 34	2.1–3.2	B8	......	*d*	I	1
1713	β Ori	5 9.7	− 8 19	0.34	B8p	......	g	I	1
1791	β Tau	5 20.0	+28 31	1.78	B8	......	d	I	1
3623	κ Cnc	9 2.3	+11 4	5.14	B8	......	*d*	I	1
4662	γ Crv	12 10.7	−16 59	2.78	B8	......	d	I	1
6812	μ Sgr	18 7.8	−21 5	4.01	B8p	......	g	II	1
7906	α Del	20 35.0	+15 34	3.86	B8	......	*d*	II	1
8541	4 Lac	22 20.5	+48 58	4.64	B8p	......	g	I	1
8634	ζ Peg	22 36.5	+10 19	3.61	B8	......	*d*	II	1
8965	17 And	23 33.2	+42 43	4.28	B8	......	*d*	II	1
223	ν Cas	0 43.2	+50 25	5.03	B9	......	*d*	*I*	1
1204	+62°628	3 48.6	+62 47	4.87	B9	......	*d*	I	1
4116	δ Sex	10 24.4	− 2 14	5.24	B9	......	*d*	II	1
5982	υ Her	15 59.7	+46 19	4.64	B9	......	*d*	I	1
7236	λ Aql	19 0.9	− 5 2	3.55	B9	......	*d*	*II*	1
7750	κ Cep	20 12.3	+77 25	4.40	B9	......	*d*	*I*	1

The number of spectra on each kind of emulsion, and a mean value for the weakest line measurable for each of the kinds principally used, are given in Table II.

Lines fainter than could be measured by the method about to be described could be seen on the Process plates of sharp-line stars. In the wide-line spectra, however, especially those on fast grainy plates, the situation was reversed; lines could be measured by the present method that could neither be found visually nor measured satisfactorily on tracings. This feature has made possible the study of many wide-line spectra in stars too faint to permit the securing of Process spectrograms.

The plates were chosen so that every star of type B9 or earlier for

which a well-guided, adequately exposed plate was available was represented by at least one spectrogram. The total absorptions were measured on a photoelectric microphotometer[10] belonging to Dr. H. Rosenberg. In this photometer, a microscope objective projects a reduced image of a uniformly illuminated diaphragm (square, in this case) on the plate being measured. The diaphragm image must be large enough to include all of the line being measured. A second microscope objective produces an image of this region of the plate on the mirror of a Lummer-Brodhun cube, whence almost all the

TABLE II

PHOTOGRAPHIC EMULSIONS USED

Emulsion	No. of Spectra	Weakest Line Measurable
Eastman 40	34	0.08 A
Eastman Process	30	.04
Eastman Hyper-press	13	0.06
Imperial Eclipse Soft	3	....
Eastman IV-O Spectrographic	1	....
Eastman Supersensitive Portrait Film	1	....

light passes through a neutral wedge to the photocell. An electrometer is used to indicate balance between the output of this cell and that of another photocell, illuminated by the same lamp, which is used for compensation, the position of the neutral wedge being varied until this balance occurs. The setting of the wedge can easily be read to 0.1 mm, or about 0.001 mag. Repeated measures on one point of a plate are useless; the steadiness of the electrical circuits of this instrument is such that the readings would all be the same.

The procedure in measuring was as follows: the spectrum was first oriented parallel to one screw of the plate carriage, by inspection through the setting eyepiece. This eyepiece, focused on the mirror of the Lummer-Brodhun prism, gives the operator a view not only of the portion of the plate illuminated by the measuring beam but of the neighboring portion of the plate as well; this feature made

[10] For a more complete description of the instrument than is given below, see *Zs. f. Instrumentenkunde*, **45**, 313, 1925, or *Handb. der Astroph.*, **2**, Part 1, p. 425, 1929.

easy such things as the orientation mentioned, the recognition of the spectral region, and the avoidance of bad flaws or scratches on the plate. When the plate had been oriented and the sensitivity of the electrometer adjusted to cover the range of densities needed, five (sometimes more) wedge readings were taken for each line, as follows: one with the line centered in the diaphragm; two on clear film opposite the line (one above and one below the comparison spectrum); and two on the continuous spectrum, at either side of the line. Where the density of the continuous spectrum was changing fast, two places on each side were measured. From this last group, a reading was deduced for the continuous spectrum at the position of the line. The mean of the clear film readings was used in the reductions as the zero point of that line. In addition to measures on the spectrum, three points near the center of each of the measurable spots imprinted by the tube sensitometer on each spectrogram were measured, and clear film readings were made on either side of each spot.

From the difference in wedge reading (clear film − spot) for each sensitometer spot, plotted against the known magnitude differences between spots, a characteristic curve was drawn for each plate. By means of this curve, the interval in wedge reading between line and continuous background at the position of the line was converted into a magnitude difference, and the total absorption for each line was deduced.

Table III gives the measured intensities, in angstroms of total absorption, for each line measured in more than one or two stars. As will be shown later, the values given for the hydrogen lines are not actual total absorptions, but they are consistent fractions of the total absorptions regardless of broadening due to rotation or to Stark effect. Included also in Table III, in italics, are eye estimates of the total absorptions of some lines which were not measurable, because of weakness, or plate flaws. These estimates were calibrated by means of the measured intensities of other lines in the same spectrum or in other spectra of similar type and line width. An entry in the table such as < 0.08 indicates that the line was absent (that is, it could neither be seen nor measured) and that the weakest line shown with any certainty on that plate had an intensity of approxi-

TABLE III

TOTAL ABSORPTIONS*

Wave-Length	ξ Per Oe5	λ Ori Oe5	θ¹ Ori Oe5	Bond 640 Oe5	ι Ori Oe5	S Mon Oe5	29 CMa Oe	τ CMa Oe5	19 Cep Oe5	10 Lac Oe5
He I										
4472	0.59	0.61	0.52	0.91	0.87	0.51	0.70	0.96	0.97	0.81
4026	0.58	0.59	0.48	1.05	0.60	0.49	0.55	0.65	0.53	0.64
4713	0.31	0.13	0.28	0.13:	0.22	0.16	0.34	0.23	0.33	0.23
4121	0.09	0.16	0.12	0.34	0.15	0.08	0.05	0.18	0.14	0.13
4388	0.20	0.21	0.27	0.65	0.34	0.21	0.16	0.59	0.20	0.32
4144	*0.12*	0.11	0.13	0.57	0.22	0.14	*0.08*	0.41	0.16	0.19
4009	0.06	0.06	0.05	0.46	0.15	*0.04*	0.02:	0.18	0.02	0.12
3965	.....	.....	0.19	.....	.....	.....	.....	.....	.....	.....
4438	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4169	.....	0.03	.....	.....	0.04	.....	.....	.....	.....	0.03
He II										
4200	0.33	0.28	0.48	*0.10*	0.26	0.33	0.24	0.25	*0.15*	0.18
4542	0.55	0.53	0.28	<*0.10*	0.42	0.44	0.48	0.39	0.30	0.46
4686	0.32	0.40	0.60	0.25	0.44	0.49	emis.	0.26	0.22	0.71
H										
4102	1.22	1.37	1.31†	2.18†	1.42	1.16	1.09	1.68	1.32	1.09
4340	1.51	1.38	1.51†	2.85†	1.81	1.39	1.11	1.79	1.36	1.57
4861	1.78	.....	.....	.....	.....	.....	.....	1.52:	.....	.....
Si IV										
4089	*0.10*	0.22	0.04	*0.18*	0.24	0.12	0.14	0.39	0.43	0.22
4116	*0.06*	0.16	0.04	*0.12*	0.12	0.04	0.13	0.34	0.32	0.11
4212	.....	0.07	.....	.....	0.12	.....	.....	.....	.....	0.04
Si III										
4552	.....	.....	.....	.....	0.19	.....	.....	.....	0.20:	0.08
4567	.....	.....	.....	.....	.....	.....	.....	.....	0.16:	0.04
4574	.....	.....	.....	.....	.....	.....	.....	.....	0.16:	0.03
Si II										
4128	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4131	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
N III										
4097	*0.15*	0.19	0.08	*0.10:*	0.16	0.15	0.54	0.24	0.37:	0.31
4379	.....	0.07	.....	.....	0.09	0.06	.....	0.09	.....	0.07
N II										
3995	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4237	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4242	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Mg II										
4481	*0.12*	0.06	*0.05*	<*0.10*	0.10	0.07	*0.05*	0.08	0.11	0.05
C II										
4267	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

* Intensities are in angstroms of total absorption. Italics indicate estimated values. Hydrogen intensities are not actual total absorptions; see text.

† These values were determined from tracings.

TABLE III—*Continued*

Wave-Length	κ Cas B0	α Cam B0	δ Ori B0	ϕ^1 Ori B0	ε Ori B0	ζ Ori B0	κ Ori B0	ρ Leo B0p	τ Sco B0	ζ Per B1
He I										
4472	0.66	1.19	0.75	1.29	0.84	1.20	1.04	0.91	1.07	0.95
4026	0.58	0.86	0.46	0.78	0.56	0.74	0.62	0.68	0.71	0.61
4713	0.19	0.34:	0.25	0.53	0.19	0.25	0.35	0.34	0.17	0.39
4121	0.41:	0.25	0.15	0.36	0.27	0.20	0.27:	0.38	0.25	0.30:
4388	0.45	0.30	0.25	0.72	0.38	0.36	0.44	0.41	0.60	0.47
4144	0.30	0.20	0.15	0.57	0.23	0.26	0.28	0.33	0.37	0.37
4009	0.29	0.13	0.11	0.29	0.15	0.12	0.17	0.31	0.23	0.30
3965	...	...	...	0.12	...	0.13	...	...	...	0.21:
4438	...	...	...	...	...	...	...	...	...	...
4169	...	...	0.01	...	0.03	0.04	0.06	0.08	0.02	...
He II										
4200	*<0.08*	0.23	0.13	*0.04:*	0.10	0.24	0.06	*0.05:*	0.07	...
4542	*0.10*	0.26	0.17	*0.02:*	0.14	0.33	0.04	*<0.10*	0.15	...
4686	*0.10*	0.05:	0.23	0.52	0.08:	0.18	0.20	*<0.10*	0.27	0.02:
H										
4102	0.99	1.71:	1.03	1.50	1.11	1.52	1.12	0.98	1.48	1.08
4340	1.10	1.36	1.25	2.05	1.21	1.82	1.54	1.53	2.09	1.55
4861	...	...	...	...	...	...	...	...	...	...
Si IV										
4089	0.27	1.00	0.32	0.26	0.62	0.38	0.43	0.18	0.09	0.09
4116	0.19	0.57	0.21	0.20	0.50	0.35	0.28	0.08	0.06	0.03
4212	...	...	...	...	0.04	...	...	...	0.03	...
Si III										
4552	0.64	*0.15:*	0.10	0.54	0.32	0.22	0.36	0.55	0.20	0.45
4567	0.50	*0.10*	0.06	0.48	0.24	0.13:	0.23	0.50	0.17	0.37
4574	0.29	*<0.10*	0.04	0.27	0.14	0.10:	0.13	0.29	0.14	0.18
Si II										
4128	...	...	...	...	...	...	...	...	...	...
4131	...	...	...	...	...	...	...	...	...	...
N III										
4097	*0.08*	0.87	0.15	0.13	0.37	0.29	0.26	0.10	0.20	0.11:
4379	...	0.25	...	...	0.06	0.03	0.04	...	0.07	...
N II										
3995	...	...	...	...	...	...	...	...	...	0.22
4237	...	...	...	...	...	...	...	...	...	...
4242	...	...	...	...	...	...	...	...	...	...
Mg II										
4481	0.06	*0.10:*	0.02	0.15	0.08	0.15	0.15	0.27	0.11	0.20
C II										
4267	...	...	...	0.18	...	...	...	...	...	0.19

TABLE III—*Continued*

Wave-Length	ε Per B1	β CMa B1	ε CMa B1	β Sco B1	γ Peg B2	δ Cet B2	γ Ori B2	χ^2 Ori B2p	9 Cep B2p	ζ Cas B3
He I										
4472....	1.41	0.90	0.78	0.91	1.42	1.36	1.39	0.90	1.00	1.42
4026....	1.07	0.98	*0.75:*	0.49	0.98	1.13	1.19	0.66	0.57	1.00
4713....	0.32	0.32	0.29	0.15	0.31	0.43	0.51	0.25	0.42	0.31
4121....	0.42	0.38	*0.12:*	0.23	0.34	0.46	0.39	0.30	0.34	0.34
4388....	0.80	0.62	0.53	0.42	0.92	0.91	0.83	0.73	0.48	0.97
4144....	0.53	0.46	*0.30:*	0.31	0.69	0.76	0.72	0.40	0.33	0.78
4009....	0.40	0.44	*0.30:*	0.21	0.45	0.62	0.56	0.40	0.26	0.52
3965....	0.49:	0.28	......	0.23:	0.26	0.19	0.23	0.48	0.29	0.20:
4438....	......	0.09	0.12	......	0.09	......	......	......	0.01	0.08
4169....	......	0.05	......	0.05	0.07	......	0.10	......	0.04	0.12
He II										
4200....	......	......	......	......	......	......	......	......	......	......
4542....	......	......	......	......	......	......	......	......	......	......
4686....	......	......	......	0.04	......	......	......	......	......	......
H										
4102....	1.79	1.56	......	1.41	1.82	2.05	2.13	1.14	1.07	1.74
4340....	2.12	1.52	1.19:	1.58	2.32	2.46	2.96	1.27	1.33	2.25
4861....	......	1.82	......	......	......	......	2.71	......	......	......
Si IV										
4089....	*0.20:*	0.11	*0.06:*	0.08:	0.02	......	......	......	......	......
4116....	0.07:	0.01:	*0.03:*	0.02	......	......	......	......	0.01	......
4212....	......	......	......	......	......	......	......	......	......	......
Si III										
4552....	*0.25*	0.36	0.45	0.18	0.20	0.13	0.21	0.58	0.45	0.18
4567....	*0.15*	0.30	0.34	0.13	0.15	0.06	0.21	0.44	0.32	0.14
4574....	*0.10*	0.14	0.21	0.10	0.10	0.06	0.16	0.42	0.19	0.09
Si II										
4128....	......	......	......	......	......	......	......	......	0.09	0.01
4131....	......	......	......	......	......	......	......	......	0.15	0.03
N III										
4097....	*0.10:*	*0.02*	*0.02:*	*0.03*	......	......	......	......	......	......
4379....	......	......	......	0.01	......	......	......	......	......	......
N II										
3995....	*0.30*	0.18	*0.20:*	*0.08*	0.06	0.08	0.11	0.54	0.16	0.06
4237....	......	0.03	......	......	0.01	......	......	......	0.03	......
4242....	......	0.07	......	......	0.04	......	......	......	0.04	......
Mg II										
4481....	0.31	0.18	0.20	0.14	0.24	0.28	0.21	0.28	0.34	0.32
C II										
4267....	*<0.15*	0.17	0.26	0.08	0.17	0.24	0.25	0.16	0.22	0.20

TABLE III—*Continued*

Wave-Length	π And B3	ε Cas B3	λ Tau B3	η Aur B3	31 Crv B3	η UMa B3	u Her B3	θ Oph B3	ι Her B3	102 Her B3
He I										
4472....	0.84	0.81	0.93	1.06	0.85	1.08	1.80	1.13	1.18	1.14
4026....	0.72	0.64	0.85	0.83	0.80	0.78	0.99	1.13	1.00	0.84
4713....	0.13	0.17	0.15	0.27	0.35	0.08:	0.52	0.31	0.38	0.24
4121....	0.04:	0.11	0.14	0.17	0.23	0.11	0.36	0.41	0.20	0.25
4388....	0.54	0.47	0.45	0.59	0.42	0.64	1.11	0.71	0.82	0.81
4144....	0.45	0.42	0.46	0.55	0.54	0.47	1.17	0.59	0.66	0.58
4009....	0.31	0.25	0.36	0.33	0.37	0.31	0.45	0.54	0.36	0.47
3965....	......	......	0.45:	0.24	......	......	......	......	0.36	0.33
4438....	0.04	0.03	0.12	0.12	......	0.09	......	0.03	0.14	0.08
4169....	......	0.03	0.19	0.03	0.09	0.06	0.25	0.12	0.09	0.08
He II										
4200....	......	......	......	......	......	......	......	......	......	......
4542....	......	......	......	......	......	......	......	......	......	......
4686....	......	......	......	......	......	......	......	......	......	......
H										
4102....	2.60	1.88	2.14	2.27	2.17	2.00	2.84	1.87	2.22	1.51
4340....	3.28	2.46	2.51	2.93	2.17	2.58	3.39	2.38	2.96	2.00
4861....	......	......	......	......	......	......	......	......	......	......
Si IV										
4089....	......	......	......	......	......	......	......	0.05	......	......
4116....	......	......	......	......	......	......	......	......	......	......
4212....	......	......	......	......	......	......	......	......	......	......
Si III										
4552....	0.05	0.04:	0.20:	*<0.06*	0.37	0.07:	0.46	0.21	0.16	0.15
4567....	0.04:	*0.02*	0.10:	*<0.06*	0.29	0.04:	0.29	0.13	0.12	0.11
4574....	*0.03:*	*0.02*	0.04:	*<0.06*	0.17	0.02:	0.14	0.13	0.10	0.07
Si II										
4128....	*0.04*	0.06	*0.08*	0.02	*0.05:*	0.04	0.06	*<0.08*	0.07	0.01:
4131....	*0.04*	0.08	*0.06*	0.05	*0.05:*	0.06	0.12	*<0.08*	0.09	0.02:
N III										
4097....	......	......	......	......	......	......	......	......	......	......
4379....	......	......	......	......	......	......	......	......	......	......
N II										
3995....	*0.06*	*0.02:*	*0.15*	0.05	*0.10*	0.01:	*0.10:*	0.05	0.02	0.08
4237....	......	......	......	......	......	......	......	......	......	0.04:
4242....	......	......	......	......	......	......	......	......	......	0.06:
Mg II										
4481....	0.31	0.24	0.44	0.27	0.21	0.35	0.67	0.20	0.26	0.20
C II										
4267....	0.14	0.15	0.17	0.14	0.11	0.15	0.34	0.09	0.20	0.20

TABLE III—*Continued*

Wave-Length	η Lyr B3	σ Aql B3	1H. Cas B3	α Scl B5	20 Tau B5	ν Gem B5	γ CMa B5	27 CMa B5p	τ Her B5	ζ Dra B5
He I										
4472....	0.88	1.28	1.32	0.37	0.55	0.77	0.27	1.69	0.88	0.47
4026....	0.76	0.70	1.01	0.38	0.37	0.51	0.27	1.35	0.82	0.41
4713....	0.23	0.29	0.51:	0.21	0.13:	0.27	*0.15:*	0.48	0.21	0.18
4121....	0.19	0.12	0.14	0.04	0.06	*0.15:*	0.06:	0.33	0.08	0.03
4388....	0.52	0.81	0.71	0.21:	0.35	0.54	0.29:	0.77	0.66	0.28
4144....	0.46	0.65	0.55	0.28	0.21	0.39	0.14	0.83	0.35	0.22
4009....	0.42	0.30	0.39	0.07	0.08	0.18	0.10	0.57:	0.24	0.06
3965....	0.17	......	......	......	......	......	......	......	......	.....
4438....	0.09	0.10	......	......	......	......	......	......	......	.....
4169....	0.07	0.04	......	......	......	......	......	......	......	.....
He II										
4200....	......	......	......	......	......	......	......	......	......	.....
4542....	......	......	......	......	......	......	......	......	......	.....
4686....	......	......	......	......	......	......	......	......	......	.....
H										
4102....	1.89	1.78	2.44	2.64	2.43	2.13	2.80	2.49	2.75	2.45:
4340....	2.31	2.56	2.81	3.24	3.28	2.70	3.18	3.12	3.36	3.22:
4861....	......	......	......	4.02	......	......	4.19	......	......	.....
Si IV										
4089....	......	......	......	......	......	......	......	......	......	.....
4116....	......	......	......	......	......	......	......	......	......	.....
4212....	......	......	......	......	......	......	......	......	......	.....
Si III										
4552....	0.08	0.38	0.13	......	......	......	......	0.24:	......	0.07:
4567....	0.04	<*0.25*	0.13:	......	......	......	......	......	......	.....
4574....	0.01	<*0.25*	0.09:	......	......	......	......	......	......	.....
Si II										
4128....	0.07	*0.05*	*0.04:*	0.09	0.06	0.18	0.08	<*0.20*	0.08	0.11
4131....	0.09	*0.05*	*0.04:*	0.14	0.10	0.15	0.08	<*0.20*	0.08	0.12
N III										
4097....	......	......	......	......	......	......	......	......	......	.....
4379....	......	......	......	......	......	......	......	......	......	.....
N II										
3995....	0.04	0.07	0.06	......	......	......	......	......	......	.....
4237....	......	......	......	......	......	......	......	......	......	.....
4242....	......	......	......	......	......	......	......	......	......	.....
Mg II										
4481....	0.26	0.27	0.39	0.25	0.31	0.43	0.32	0.16	0.33:	0.31
C II										
4267....	0.16	0.21	0.10	0.17	0.12	*0.10:*	*0.08:*	*0.10*	0.12	0.05

TABLE III—*Continued*

Wave-Length	67 Oph B5p	ε Del B5	λ Cyg B5	2 Lac B5	β Per B8	β Ori B8p	β Tau B8	κ Cnc B8	γ Crv B8	μ Sgr B8p
He I										
4472	0.62	0.44	0.74	0.69	0.78	0.38	0.48	0.30	0.19	0.33
4026	0.62	0.43	0.70	0.51	0.61:	0.29	0.40	0.25	0.25:	*0.30:*
4713	0.32	0.22	0.44	0.08	0.54:	0.21	*0.07:*	*<0.05*	*0.07:*	*<0.05*
4121	0.16	0.05	0.12	0.09	0.10	0.08	0.05	*0.03*	0.03	0.06:
4388	0.47	0.34	0.50	0.27	0.20	0.36	0.26	0.14	0.15	0.33
4144	0.34	0.21	0.35	0.26	0.15	0.14	0.19	*0.08*	0.07:	0.25
4009	0.26	0.08	0.28	0.16	0.18	0.15	0.10	0.03:	0.05	*0.20:*
3965	0.14	...	0.43:	...	...	0.18:	...	...	...	...
4438	0.06	...	...	...	...	...	...	...	...	...
4169	0.02:	0.02:	0.06	...	...	0.04:	...	...	...	...
He II										
4200	...	...	...	...	...	...	...	...	...	...
4542	...	...	...	...	...	...	...	...	...	...
4686	...	...	...	...	...	...	...	...	...	...
H										
4102	1.72	2.55	2.85	2.46	3.68	1.09	2.48	3.04:	2.60	1.36
4340	2.22	2.76	3.32	3.05	4.31	1.15	3.22	3.67	3.23	1.38
4861	...	...	...	...	5.95	1.47	...	5.14:	...	...
Si IV										
4089	...	...	...	...	...	...	...	...	...	...
4116	...	...	...	...	...	...	...	...	...	...
4212	...	...	...	...	...	...	...	...	...	...
Si III										
4552	0.12	0.04:	0.07	...	...	...	...	...	...	...
4567	0.10	...	0.04:	...	...	...	...	...	...	...
4574	0.07	...	...	...	...	...	...	...	...	...
Si II										
4128	0.13	0.08	0.15:	0.07	0.15	0.15	0.15	0.14	0.11	0.28
4131	0.15	0.06	0.12:	0.07	0.10	0.15	0.12	0.14	0.10	0.33
N III										
4097	...	...	...	...	...	...	...	...	...	...
4379	...	...	...	...	...	...	...	...	...	...
N II										
3995	0.05	...	0.14	...	...	0.07:	...	...	...	...
4237	...	...	...	...	...	...	...	...	...	...
4242	...	...	...	...	...	...	...	...	...	...
Mg II										
4481	0.42	0.24	0.34	0.42	0.64	0.44	0.30	0.31	0.22	0.48
C II										
4267	0.26	0.08	0.18	0.16	*<0.08*	0.13	0.04	*0.08*	0.03	*0.10:*

TABLE III—*Continued*

Wave-Length	α Del B8	4 Lac B8p	ζ Peg B8	17 And B8	ν Cas B9	+62° 628 B9	δ Sex B9	υ Her B9	λ Aql B9	κ Cep B9
He I										
4472	0.34	0.26	0.29	0.26	0.40	0.15	0.27	0.18	0.27	0.22
4026	0.31	0.34:	0.18	0.30	0.32	0.20	0.24	0.31	0.20	0.15
4713	0.26	*0.05*	*0.04:*	0.17	0.05:	*0.10*	<*0.10*	*0.06*	*0.15:*	*0.08*
4121	0.09:	0.03:	*0.03*	0.06:	*0.05*	<*0.07*	*0.10*	*0.05*	<*0.10*	0.10:
4388	0.30	0.24:	0.18	0.25	0.09:	0.14	0.26	0.14	0.25	0.29:
4144	0.14	0.23	0.05	0.13	0.21	0.02:	0.14	0.06	0.07:	0.08:
4009	0.06:	*0.05*	0.02:	0.08	*0.03:*	*0.07*	<*0.10*	*0.04*	<*0.10*	*0.08*
3965	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4438	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4169	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
He II										
4200	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4542	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4686	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
H										
4102	3.34	2.41	2.72	3.02	3.27	3.16:	3.15:	flaw	3.53:	3.67
4340	4.32	2.73	3.20	3.79	4.00	4.10	4.05	3.88	4.53	4.55
4861	5.56	.....	.....	5.16	8.83:	5.48:	.....	4.84:	5.74	5.66
Si IV										
4089	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4116	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4212	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Si III										
4552	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4567	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4574	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Si II										
4128	0.07	0.18	0.06	0.20	*0.04:*	0.07	*0.09*	0.08	<*0.10*	0.18
4131	0.10	0.23	0.06	0.19	*0.04:*	0.14	*0.09*	0.08	<*0.10*	0.13
N III										
4097	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4379	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
N II										
3995	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4237	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4242	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Mg II										
4481	0.64	0.68	0.37	0.31	0.29	0.38	0.55	0.34	0.54	0.56
C II										
4267	*0.08*	0.08:	0.01:	0.10	.....	.....	.....	.....	.....	.....

mately 0.08 A. The colon (:) is used to indicate an uncertain measure or estimate.

II. INVESTIGATION OF ERRORS

The theoretical difference between line intensities measured in the manner described and those determined by integration of profiles has been investigated by Öhman.[11] He found that where one is working on or near the straight-line portion of a characteristic curve whose slope is 1, and where the average density of continuous spectrum and

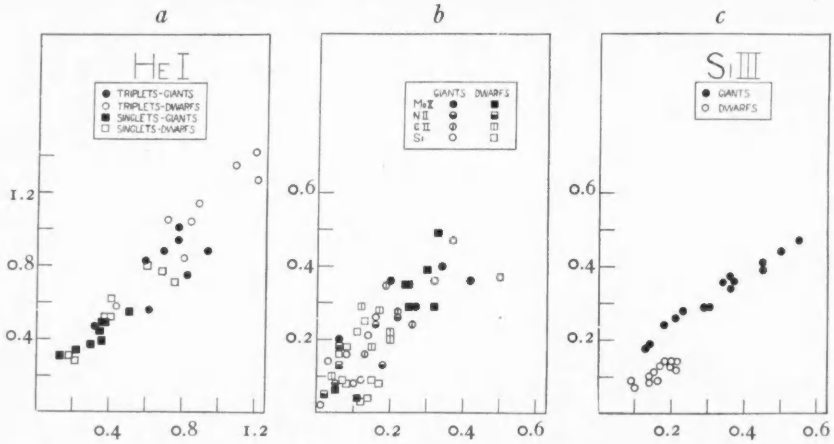


FIG. 1.—A comparison of the measured line intensities (abscissae) with (*a*) helium intensities given by Williams, (*b*) intensities of various other elements given by Williams, and (*c*) *Si* III intensities measured by Elvey. Units in both co-ordinates are angstroms total absorption.

line is 0.25 or more, the measured total absorptions should be greater than those determined from tracings by about 6 per cent. I did not reduce my intensities by this amount, however, since comparison of my results with those of Williams[1] showed a slight linear systematic difference, my values being smaller than his. These comparisons are shown in Figure 1, (*a*) and (*b*). Figure 1 (*c*), comparing my measures of lines of *Si* III with Elvey's,[7] shows no such systematic difference.

Two sets of microscope objectives were used in the photometer during this investigation. The first, used for about three-fourths of

[11] *Arkiv f. Mat. Astron. och Fysik*, **20A**, No. 23, p. 24, 1927.

the measures, gave a diaphragm image on the plate of 0.27 mm, covering 5.5 A at $H\delta$ and 7.0 A at $H\gamma$. The other set of objectives illuminated a 0.22-mm square on the plate, equivalent to 4.4 A at $H\delta$ or 5.7 A at $H\gamma$. The latter was used only on spectra having fairly sharp lines, where poor guiding or narrowness of spectrum required it. Comparison of results obtained with the two magnifications was made by measuring three plates both ways. Systematic differences were found to be small—less than 0.01 A for a line in two cases, and 0.05 A in the third. Remeasurement and reduction independent of the first set of measures, on two plates, gave for the mean accidental error a value of 0.015 A.

Systematic differences between measures on two spectrograms of the same star appear to be of a larger order, amounting to 0.10 A for the mean of ten such cases. These differences are, of course, smaller for weak lines and larger for strong ones, in absolute value; but they very seldom exceed 20 per cent of the intensity of a strong line, while for the weakest lines they may be 75 per cent. A study of these differences convinced me that measures from one excellent plate were more trustworthy than means from two or three mediocre plates; hence, only the best plate available was measured for most of the stars studied.

Another possible source of error, in the case of binaries showing two spectra, lies in the apparent shallowing of each line by the continuous spectrum from the other component. This should be serious, however, only at phases when the lines are well separated. Examination of the plates which I had measured for the four stars on my list which were known to have two spectra[12] (β Sco, σ Aql, u Her, and 2 Lac) convinced me that the lines were not visibly double for any but that of u Herculis; the microphotometer tracing of this spectrum showed that any such correction would be inappreciable. Therefore, none was made.

For the hydrogen lines, the values given in Table III for all stars are systematically too small, as the square diaphragm did not include all of the wings of these lines, even in the O-type stars. This error was investigated by means of a tracing made with the recording microphotometer on a spectrum of ι Orionis; the total absorption

[12] J. H. Moore, *Lick Obs. Bull.*, No. 355, 1924.

measured on a profile of $H\delta$ obtained from this tracing was 1.57 A; the value which I had measured with the Rosenberg photometer was 1.18 A. When the wings of the profile were cut off as the square diaphragm had cut them off, the value measured on the tracing was reduced to 1.11 A, agreeing, within 6 per cent, with the other value. The corresponding difference for $H\gamma$ was 8 per cent. The intensities given in Table III for the hydrogen lines in the two stars of the Trapezium in Orion, Bond 640 and θ^1 Orionis (Bond 628), were measured in this way also, that is, by making profiles from tracings, disregarding in these cases the sharp emission lines in the centers of the broad absorption lines, and cutting off the wings where the diaphragm would, had they been measured in the Rosenberg photometer. Judging from the case just mentioned, ι Orionis, the values so obtained should not differ appreciably from those which I would have obtained by measuring in my regular manner, if the emission had not been present.

Further comparison of results obtained by the present method with those from tracings was available from the helium and silicon intensities in three stars. For ι Orionis, seven lines were, on the average, 0.02 A weaker on the tracing than in my regular measures. For Bond 640, three lines appeared 0.08 A stronger on the tracing, in the mean. For θ^1 Orionis, λ 4472 was 0.14 A weaker on the tracing; this was a rather dense and grainy plate. Thus, there seemed to be no appreciable systematic difference between the two methods of measuring total absorptions. A tracing was also made for u Herculis, the star having the widest strong helium lines of any measured, as determined by visual inspection. From this tracing it was evident that no corrections to the total absorptions for helium were necessary, as even in this spectrum λ 4472 was not as wide as the square illuminated in the Rosenberg instrument. Therefore, all values given in Table III are to be regarded as measured total absorptions, except those given for the Balmer lines. The latter, judging from the tracings mentioned above and from Figure 2, are about two-thirds of the total absorption when the latter is 2 A, decreasing to one-third at a true value of 9 A. This figure shows a comparison of my measures of hydrogen with those by Williams[1] and by Elvey.[5] Parts (*a*) and (*b*) of this figure show, also, that there is no appreciable

systematic difference between my measures on the giant and dwarf groups of stars (see sec. III, below), as points for giants and dwarfs tend to lie along the same line, in each case. Such a difference might be expected from the relatively stronger wings of the dwarf stars, which would cause the measured intensity of a hydrogen line in a dwarf to be smaller than that in a giant, while the true total absorptions in the two are the same.

A similar systematic difference in the ratio of measured to true total absorption would be anticipated between rotating and non-

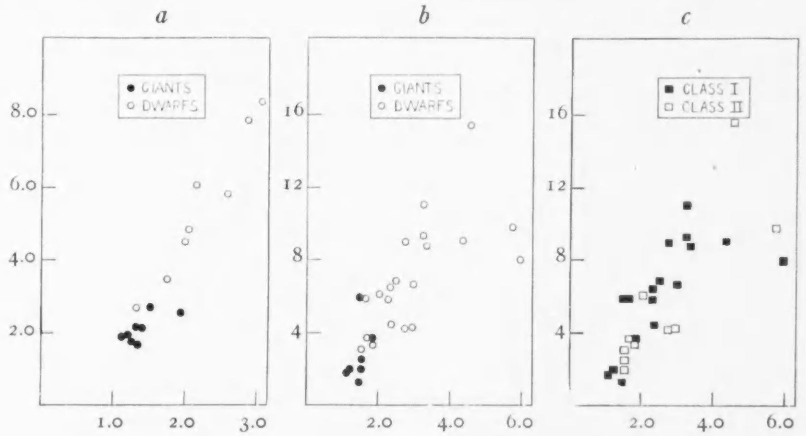


FIG. 2.—A comparison of the measured hydrogen intensities (abscissae) with (*a*) intensities measured by Williams, (*b*) intensities given by Elvey, divided according to luminosity, and (*c*) Elvey's values, divided according to line-width class. Units in both co-ordinates are angstroms total absorption.

rotating stars; the evidence of Figure 2 (*c*) would tend to show that this effect was inappreciable, since the points for rotating and non-rotating stars tend to lie along the same line. To test the matter further, the profiles given by Elvey[5] for $H\gamma$ in β Canis Majoris and in β Orionis were arbitrarily broadened by an amount corresponding to an equatorial radial velocity of 120 km/sec at the limb, keeping the total absorption constant. The intensity that would be measured by my method, which takes into account only the central 7 A of this line, was lowered by less than 3 per cent in either case, an amount well within the random errors of measurement. These profiles should

furnish a rigorous test, for the cores of these lines are almost of the same width as the image of the diaphragm used in measuring.

III. BEHAVIOR OF HYDROGEN LINES

The mean values measured for the hydrogen intensities for each Harvard spectral type are shown in Figure 3 (*a*). The slight dip from O to B0 is doubtless due to the larger proportion of dwarf stars (see below; also Table I) among the O's than among the B0's. The ratio $H\delta:H\gamma$ averages 0.83, varying between 0.8 and 0.9 for the different subtypes. Williams[1] found that the hydrogen lines he measured, from $H\gamma$ to $H\zeta$, "have nearly equal intensities."

The stars measured in each subtype were divided into giants and dwarfs, following Struve's lists[2,3] for all stars given there, and following his criteria as closely as possible for the remaining stars (italicized in Table I, third column from the right), especially as to the appearance of the hydrogen and helium lines, showing presence or absence of wings due to Stark effect. The striking difference between giants and dwarfs in the behavior of the mean of the two Balmer lines measured is shown in Figure 3 (*b*). The curves for the giants show a rise of only about 50 per cent for the stars later than B3 as compared with the earlier ones. Among the dwarfs, the rise starts earlier and is very much steeper, increasing to almost double in the interval B1–B5.

It was deemed important to search for possible systematic differences between stars such as those used by Struve in his studies, which have for the most part sharp lines, and stars whose lines are broadened appreciably by axial rotation. For this purpose the measures of rotational broadening of λ 4481 of *Mg* II and λ 4472 of *He* I made by Miss Westgate[13] were selected as an appropriate criterion, especially since her list covers all but four of the stars I measured. The other sixty-six stars I divided into two approximately equal groups: class I, including stars for which her adopted equatorial velocity of rotation is less than 65 km/sec; and class II, the stars with higher rotational velocities. The four stars not measured by her I classified by visual comparison with other stars of the same spectral

[13] *Ap. J.*, **77**, 141, 1933.

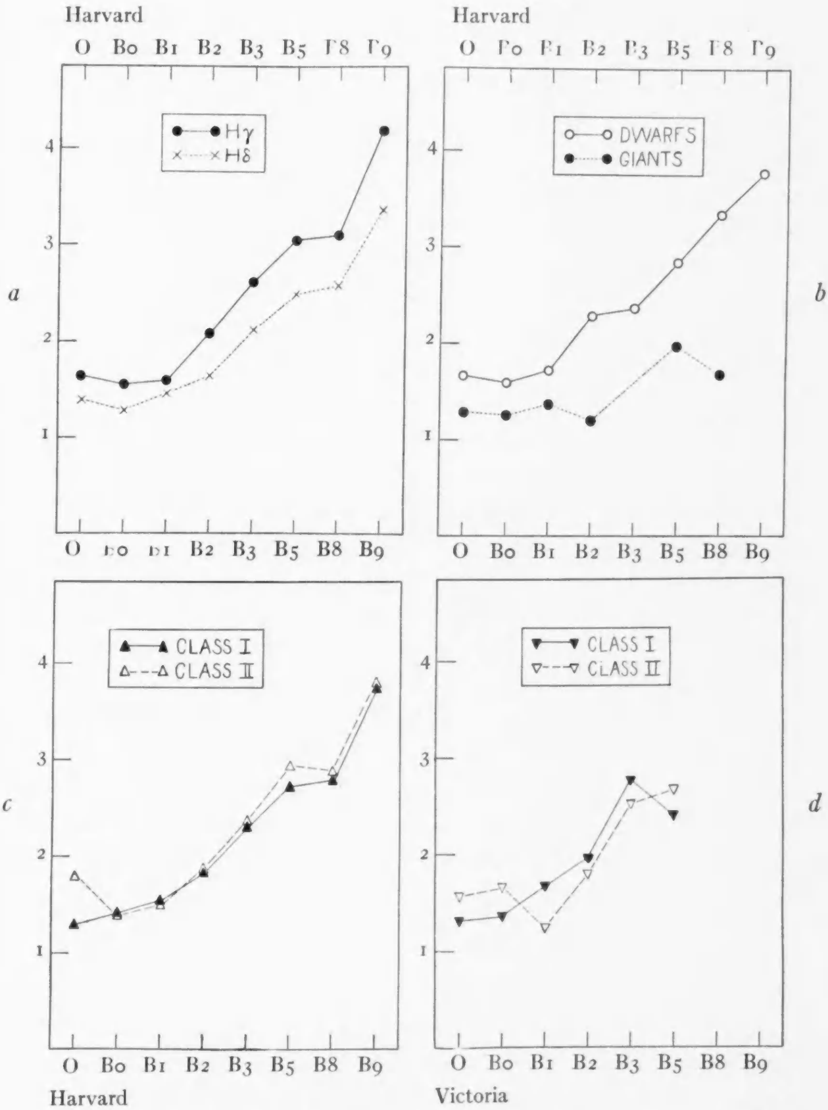


FIG. 3.—Behavior of measured hydrogen intensities with spectral type: (*a*) in all stars measured; (*b*) in giants and dwarfs, showing the mean of $H\gamma$ and $H\delta$; (*c*) in sharp-line (class I) and wide-line (class II) stars, also plotting the mean of $H\gamma$ and $H\delta$; and (*d*) similar to (*c*), except that Victoria types are substituted for Harvard. Ordinates are angstroms total absorption.

type and luminosity group. The line classes of these four are italicized in Table I.

The behavior with changing spectral type of the measured mean hydrogen intensity for the two classes of line widths is shown in Figure 3 (*c*). In type O, the apparent strengthening in the wide-line stars is partly an effect of absolute magnitude, since all four class II stars are dwarfs, while four of the six class I stars are giants. At B5, absolute magnitude again accounts for part of the difference between the two groups; the remainder may easily be due to accidental errors. It appears, then, that rotational broadening has caused no systematic difference in classification of the stars of types O and B under the Harvard system, if we take hydrogen intensities as the indicator.

As this question was found to be especially significant in connection with the behavior of neutral helium (see sec. IV, below), a further study was made, retaining the line-width class of each star but using the spectral types assigned at Victoria,[9] where this was B6 or earlier. This included forty-nine of my seventy stars; the few stars on my list classified at Victoria as B7 or later are all in class I; hence they are omitted in Figure 3 (*d*), which shows the results of this study. It might seem at first sight that the stars of class II in the types later than B0 are classified too late with respect to the sharp-line group; but upon observing the reverse effect at B4, 5, 6, and in general the large departures of both curves from smoothness, it seems to me that there is no definite systematic difference in intensity of hydrogen lines between rotating and non-rotating stars of the same Victoria spectral class.

IV. BEHAVIOR OF THE LINES OF NEUTRAL HELIUM

Figure 4 shows the behavior, with Harvard spectral type, of all the lines of neutral helium which were strong enough to be studied over the whole of the range of spectral types covered. For the sake of completeness the intensities of these seven lines were estimated in every star where they could not be measured. Where the line could not be found, its intensity was counted as zero in taking the mean for the spectral type. The dotted portion of the curve for λ 4026 indicates an allowance for blending with the *He* II line at λ 4024 in the O stars.

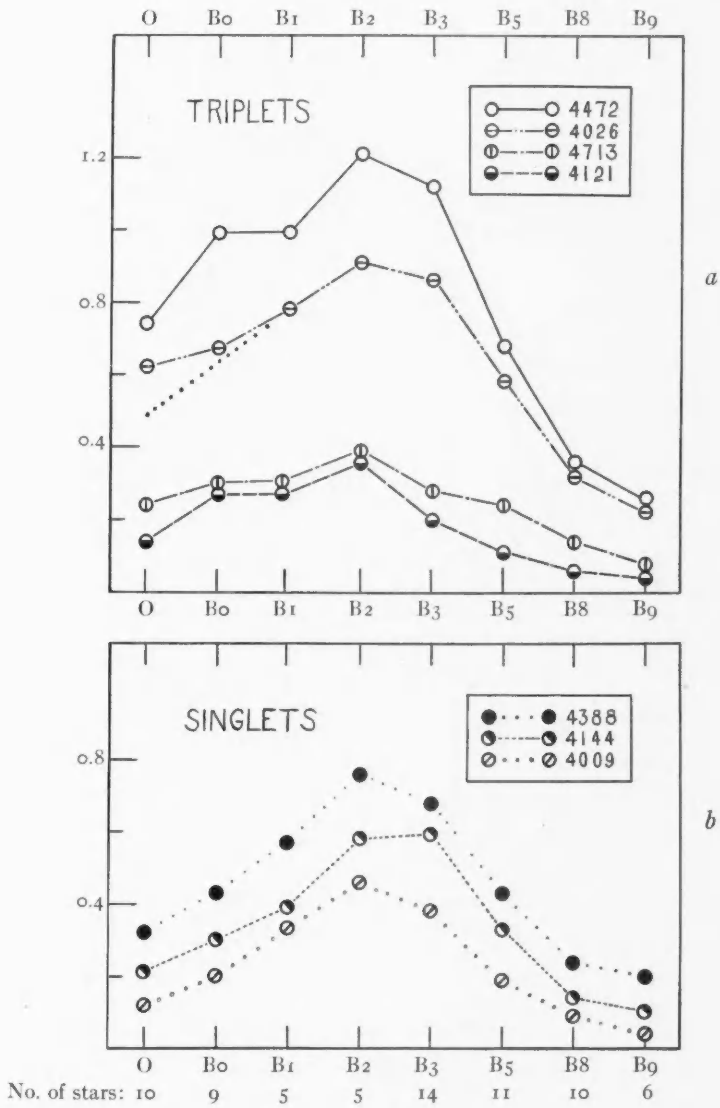


FIG. 4.—Behavior of helium intensities with spectral type, in all stars measured. Ordinates are angstroms total absorption.

The maximum at type B2 for all lines but one agrees with the results of other investigators.[14] The one exception, the singlet λ 4144, is doubtless due to accidental errors, as may be seen from the close similarity to each other, in every respect but this, of the three singlet curves. From these curves, it is obvious that there is a definite difference in behavior between the lines of the singlet and triplet systems of this element. There is evidently no displacement of maximum intensity, from one system to the other; and the behavior of the two sets of curves on the low-temperature side of the maximum is similar in many respects, but on the hot side of the maximum, in the interval from O to B2, the slopes of the curves for triplets are definitely smaller than those for the singlets. Plotted as they are, with equal intervals between Harvard subtypes, the curves for the singlets are almost symmetrical about B2.3 or thereabouts, whereas there is a definite asymmetry in every one of the triplet curves, the left portion being flatter and higher. The effect, of course, is to give a sharper, better-defined maximum for the singlets. It may be seen, also, that the triplet-to-singlet ratio (λ 4472:λ 4388 is the best example) decreases definitely from type O to type B2, and then remains sensibly constant through type B8. The results by Struve[15] and by Elvey[16] are thus confirmed on the high-temperature side of maximum.

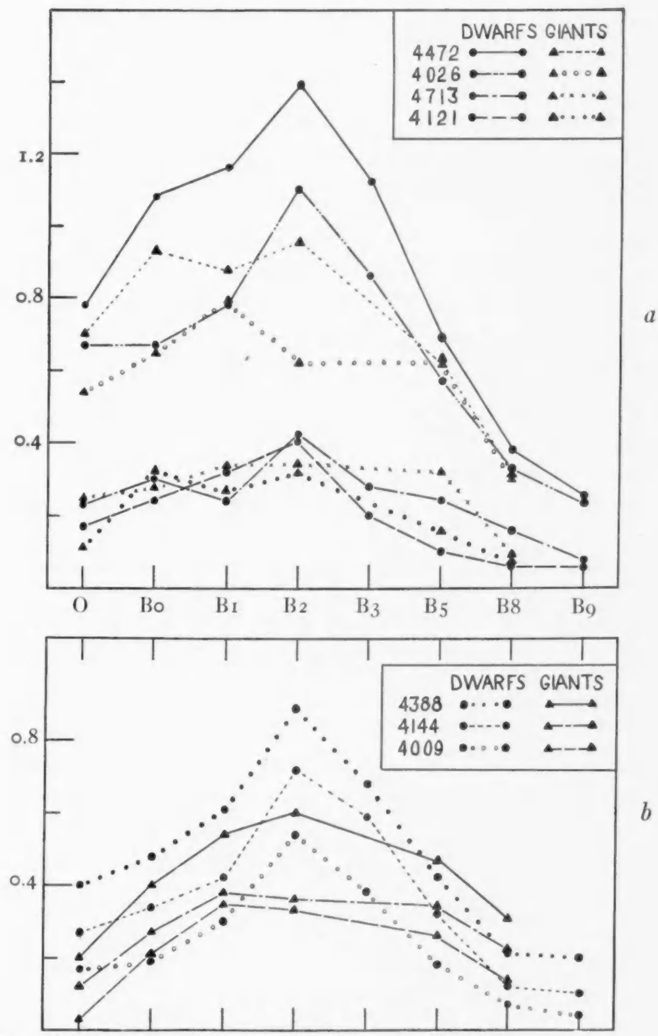
Another feature to be noted is that, as in the Balmer lines, there is a fading along each series, quite definite for the diffuse triplets in the hotter stars and for the singlets all through the interval studied.

Figure 5 shows the differences in the behavior of the helium lines between the giant and dwarf groups. In the latter, all of the lines have a well-defined sharp maximum at type B2, as has been observed by Struve, Williams, and others. The giants, on the other hand, show a flat maximum at B1, in all cases but one. It is suggested that the sharpness of the maximum in the dwarfs may be due to a tendency, in classifying, to bunch all of the stars having the strongest helium lines in one subdivision. The weakness of the hy-

[14] Cf. Miss Payne, *The Stars of High Luminosity*, p. 90, 1930; Struve, *Ap. J.*, **78**, 82, 1933; Williams, *op. cit.*

[15] *Ap. J.*, **78**, 82, 1933; also *ibid.*, **74**, 249, 1931.

[16] *Ibid.*, **70**, 150, 1929.



No. of stars:	O	Bo	B1	B2	B3	B5	B8	B9
Dwarfs:	6	4	2	3	14	10	7	6
Giants:	4	5	3	2	0	1	3	0

FIG. 5.—Behavior of helium intensities in dwarf and giant groups. (*a*) shows the triplets and (*b*) the singlet lines. Intensities are angstroms total absorption.

drogen lines in giants, allowing them to be classified earlier than dwarfs having helium lines of the same strength, may account for the difference in position of the maximum intensity between the two groups. The singlet and triplet groups of curves, for both giant and dwarf stars, retain in general the difference mentioned above, namely, that the triplets fade out more slowly on the hot side of maximum than do the singlets.

Considering the strongest and most accurately measured line in each system (again λ 4388 and λ 4472), it may be noted that the triplet line λ 4472 fades out more slowly, on either side of maximum, in giants than in dwarfs. For the singlet, λ 4388, the same is true in the later types; but in the interval B2 to O, the situation is reversed; the intensities in giants drop faster than those for dwarfs, if anything.

If these differences between singlets and triplets, depending upon luminosity, were due solely to the steeper gradient of line intensity in giants, we would expect to find the singlets intermediate in behavior, as they are in strength, between the diffuse and the sharp triplet lines. That this is not the case is best shown, perhaps, by using the stars of type O as an example. The triplets, in either series, are, on the average, about 25 per cent stronger in the dwarfs than in the giants; yet every one of the singlets is at least twice as strong in the dwarfs as in the giants. Thus, the triplet-singlet differences cannot be due entirely to the gradient effect.[17]

The differences I have found between singlets and triplets in giants and dwarfs are in essential agreement with those of Struve,[18] on which he bases his suggestion that the *He* I anomaly may be explained by large departures from thermodynamic equilibrium, the departure being greater for a giant than for a dwarf.

At type B2, both λ 4472 and λ 4388 are about 50 per cent stronger in dwarfs than in giants, and the triplet λ 4472 is 50 per cent stronger in either class than its neighboring singlet λ 4388. At type B8, λ 4472 is still a little stronger in dwarfs than in giants; the singlet, however, is actually materially stronger in the giant stars. The other two members measured in the same singlet series confirm this

[17] See Struve and Elvey, *ibid.*, **79**, 409, 1934.

[18] *Ibid.*, **82**, 257, 1935, gives a summary of Struve's results on the *He* I intensities.

fact. It may be mentioned that only in the giants of type B8, as a group, does λ 4388 become as strong as λ 4472. This is contradictory to Struve's estimated data.[19]

It is interesting at this point to interject a study of a pair of stars of the same spectral type, B8, whose luminosities are known from non-spectroscopic criteria to be markedly different from each other.

TABLE IV

TOTAL ABSORPTIONS IN β ORIONIS AND β PERSEI

Element	λ	β Ori B8 Giant	β Per B8 Dwarf
He I triplets	4472	0.38	0.78
	4026	0.29	0.61:
	4713	0.21	0.54:
	4121	0.08	0.10
He I singlets	4388	0.36	0.20
	4144	0.14	0.15
	4009	0.15	0.18
H	4102	1.09	3.68
	4340	1.15	4.31
Si II	4128	0.15	0.15
	4131	0.15	0.10
Mg II	4481	0.44	0.64
C II	4267	0.13	*<0.08*

The rotational broadening in the two stars chosen is almost identical.[13] The giant, β Orionis, is the brightest member of the Orion cluster, whose parallax has been reasonably well determined,[20] giving an absolute magnitude of −6.0 for this star. The dwarf, β Persei, has a good trigonometric parallax[21] which agrees well with the parallax computed from the eclipsing binary system.[22] The line intensities for the two stars are shown in Table IV.

Among the *He* I lines, it will be noted that the stronger "diffuse" triplets are enhanced in the dwarf (cf. Fig. 5), the weaker triplets are

[19] *Ibid.*, **74**, 249, 1931.

[20] Rasmuson, *Lund Medd.*, Ser. II, No. 26, p. 48, 1921.

[21] Schlesinger, *General Catalogue of Stellar Parallaxes*, 2d ed., 1935.

[22] Shapley, *Princeton Contr.*, No. 3, p. 83, 1915.

doubtful, and the singlets may be slightly stronger in the giant. Hydrogen is very much stronger in the dwarf (cf. Fig. 3 [*b*]), as is *Mg* II (contrary to its usual behavior at B8; see Fig. 13, below). *Si* II and *C* II are both enhanced in the giant, as will be shown to be the general case. Thus in all but one of the features enumerated, this well-established giant and dwarf differ in the same manner as do the means for the giant and dwarf groups divided according to spectroscopic criteria.[23]

It is also of interest to study the changes of line intensity with changing apparent magnitude in the brighter B0 stars of the Orion group. Rasmuson[20] lists the four stars ϵ, ζ, κ, and δ Orionis as members of the Orion cluster, for which he derives a parallax of 0".0055. These stars lie within an area in the sky about 5° by 10°. Assuming that these four stars are all at the same distance, the differences between their apparent magnitudes will of course be equal to the differences between their absolute magnitudes. Therefore, such a study might be expected to show, at least to some extent, the effects of changing luminosity on line intensity. The range in apparent magnitude, however, is only about three-fourths of a magnitude, which is rather a small fraction of the total dispersion in absolute magnitude at type B0.

Figure 6 shows the intensities of lines of various elements in these four stars, plotted against their apparent magnitudes. The lines of neutral helium and of hydrogen, shown in part (*a*) of the diagram, do not display any enhancement in the less luminous stars, although that is their general tendency at B0 (see Figs. 3[*b*] and 5). There is, however, a similarity in behavior between $H\gamma$ and λ 4472, like that shown in Figure 7 (*b*), below. On the other hand, both λ 4089 of *Si* IV and λ 4097 of *N* III, shown in part (*b*) of the figure, do show quite definitely the enhancement in the more luminous stars that is characteristic of their general behavior (see sec. V, below). The two lines of *He* II plotted in part (*c*) also show fairly typical behavior; λ 4542 shows no definite trend; it will be shown to be only slightly strength-

[23] For further evidence on the luminosities of Struve's giants and dwarfs, see Miss E. T. R. Williams (*Ap. J.*, **75**, 386, 1932), and E. G. Williams (*op. cit.*).

ened in the dwarf group, in general, at B0. λ 4686, which will be demonstrated to be more markedly strengthened in the B0 dwarfs, shows just such a strengthening in this group of stars.

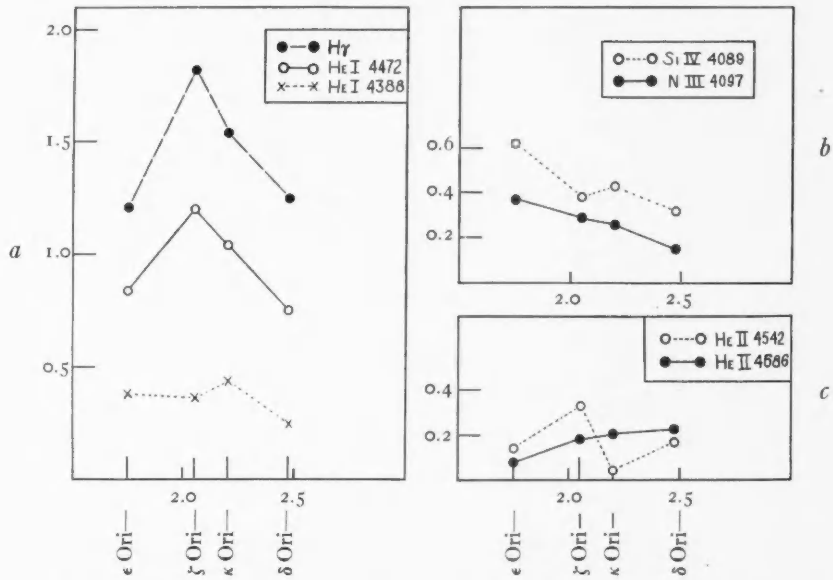


FIG. 6.—Measured intensities of various lines plotted against apparent magnitudes (abscissae) for bright stars of type B0 in Orion. Ordinates are angstroms total absorption.

In Figures 7–9, inclusive, the intensities of three singlet and three triplet lines of helium have been plotted for individual stars against the measured intensity of $H\gamma$. Each figure covers one or two Harvard subtypes, depending on the number of stars measured in each class. Thus, in each graph, the giants will be at the left, having lower hydrogen intensities. In the O stars, all the lines except perhaps λ 4472 show definite correlation of hydrogen and helium intensities, even without the isolated right-hand point (Bond 640), for which the hydrogen intensity is somewhat uncertain.

At type B0, all of the strong, well-measured lines, of both singlet and triplet systems, show the correlation mentioned above, indicating that helium lines, as well as those of hydrogen, are stronger in dwarf stars. The same is true, though less marked, in types B1 and

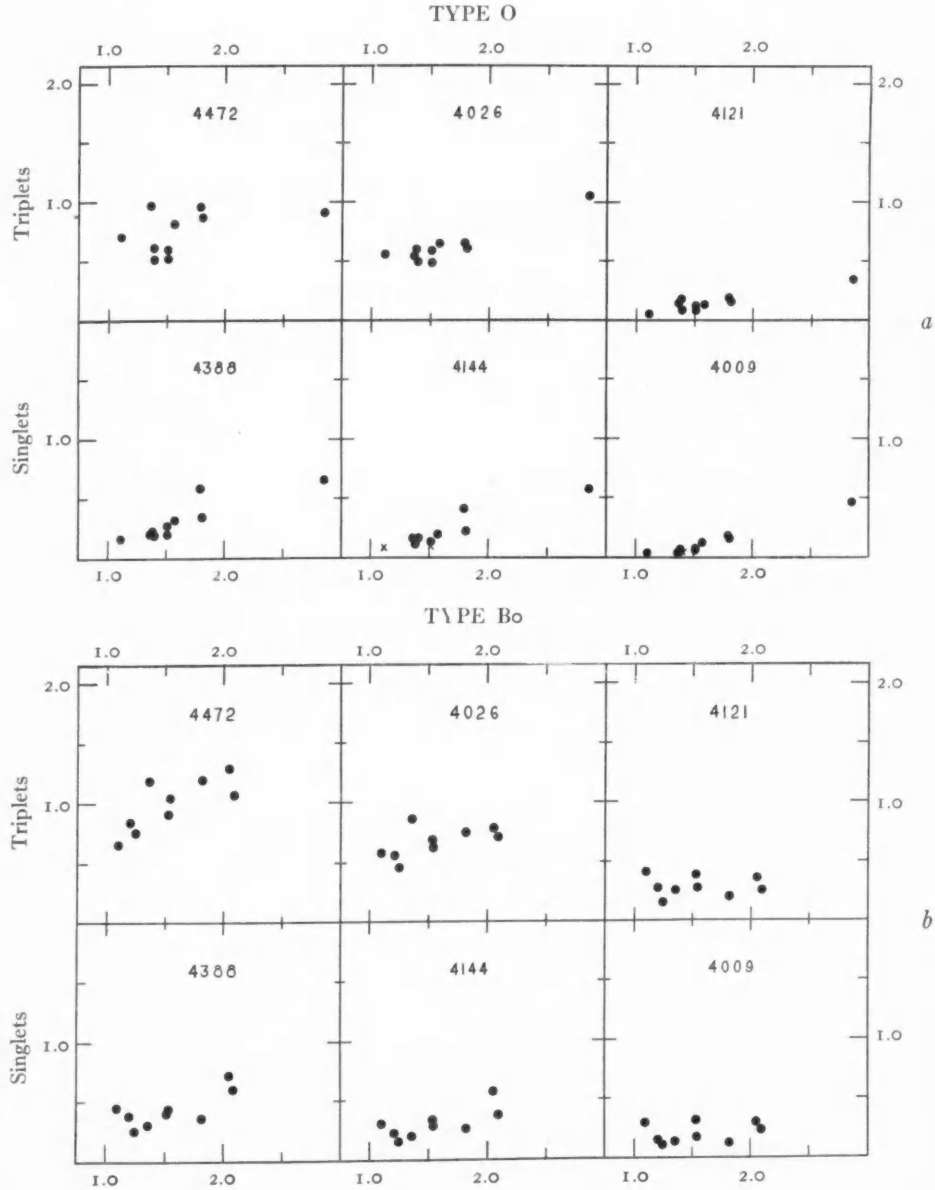


Fig. 7.—Intensities of various helium lines (ordinates) plotted against intensities of $H\gamma$ (abscissae) for stars of type O (*a*) and type Bo (*b*). All values are angstroms total absorption.

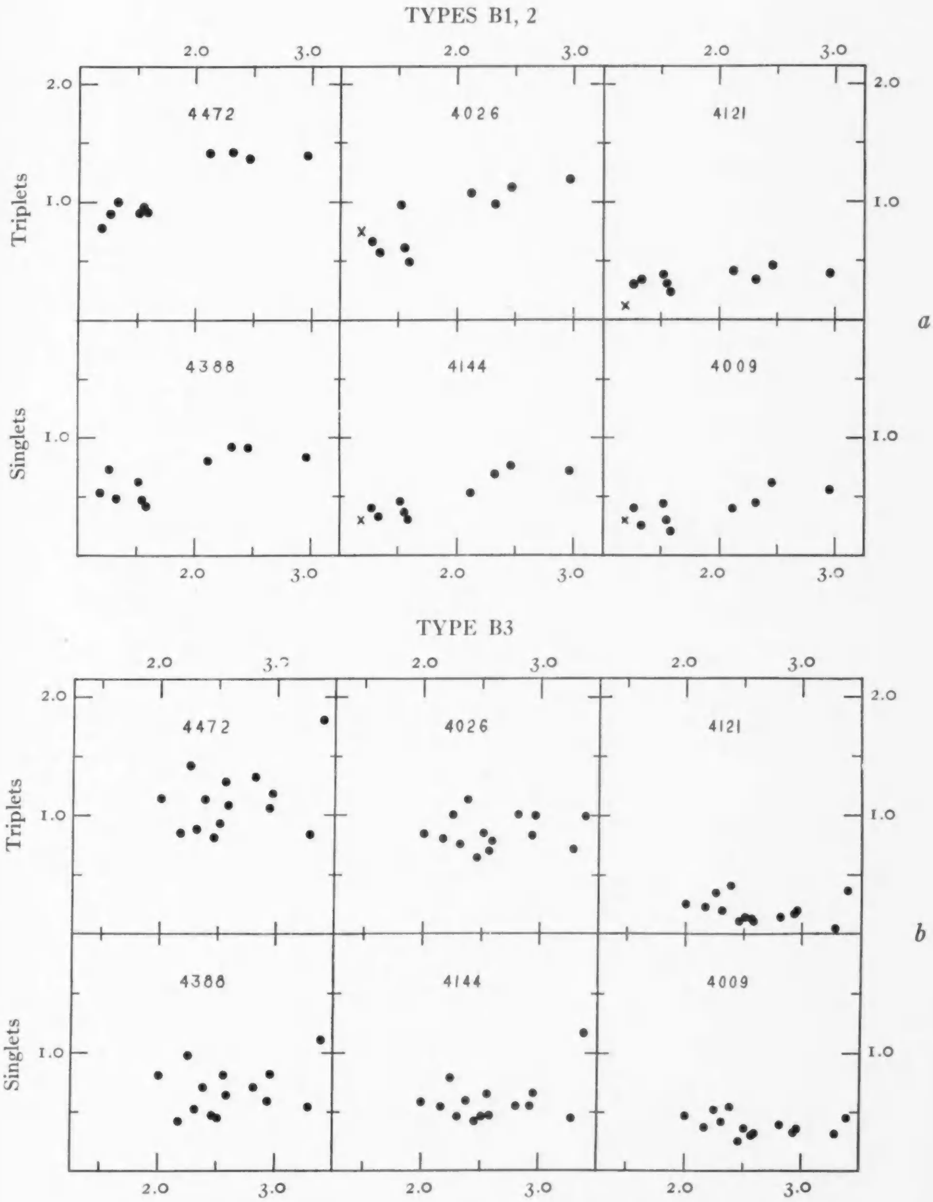


FIG. 8.—Intensities of various helium lines (ordinates) plotted against intensities of $H\gamma$ (abscissae) for stars of types B1, 2 (*a*) and type B3 (*b*). Intensities are in angstroms total absorption.

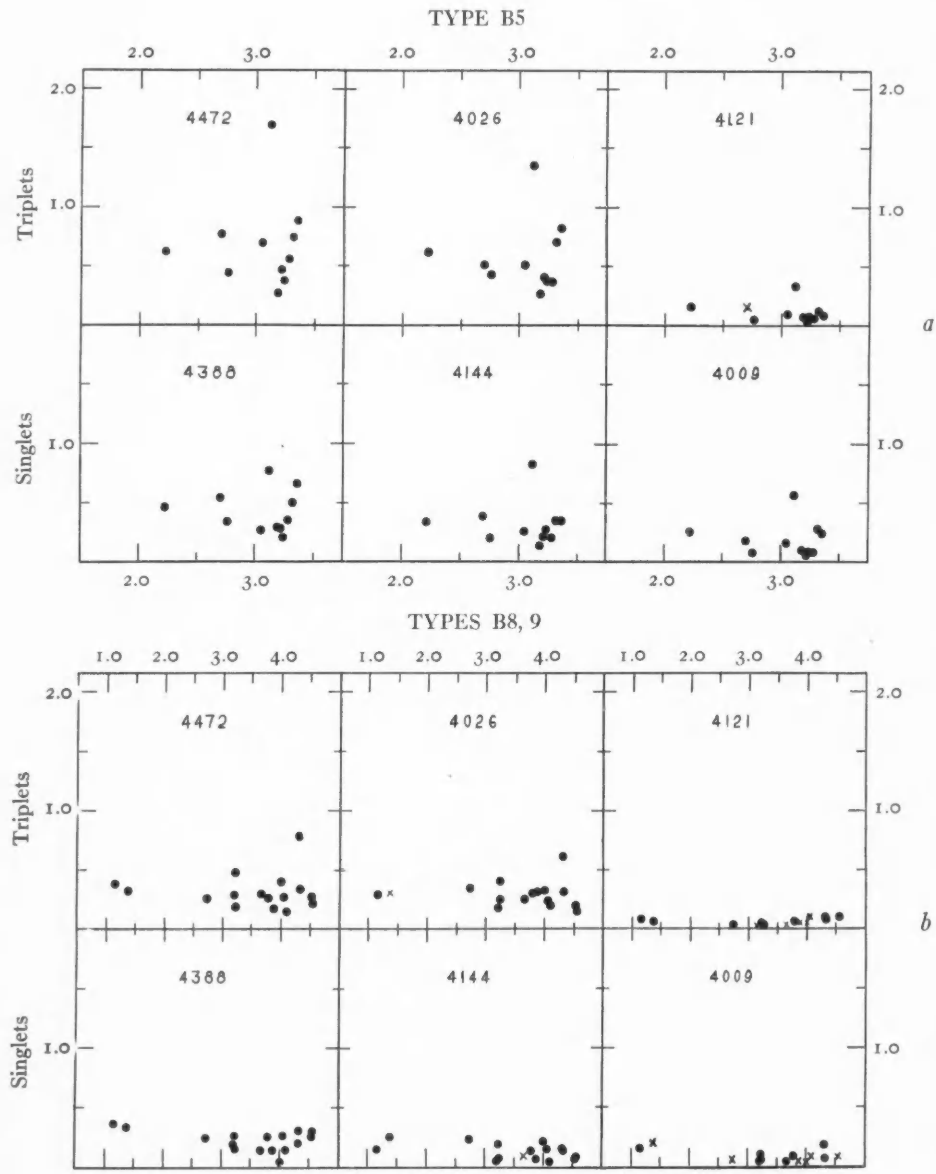


FIG. 9.—Intensities of various helium lines (ordinates) plotted against intensities of $H\gamma$ (abscissae) for stars of type B5 (*a*) and types B8, 9 (*b*). Intensities are in angstroms total absorption.

B2.[24] At B3, where none of the stars measured was called a giant, there is no appreciable correlation whatever. The graphs for type B5 and for types B8 and B9 show for some lines a suggestion of the opposite effect from that shown in the groups earlier than B3,[25] but the intensities are so small that the apparent effect may be due to errors of measurement.

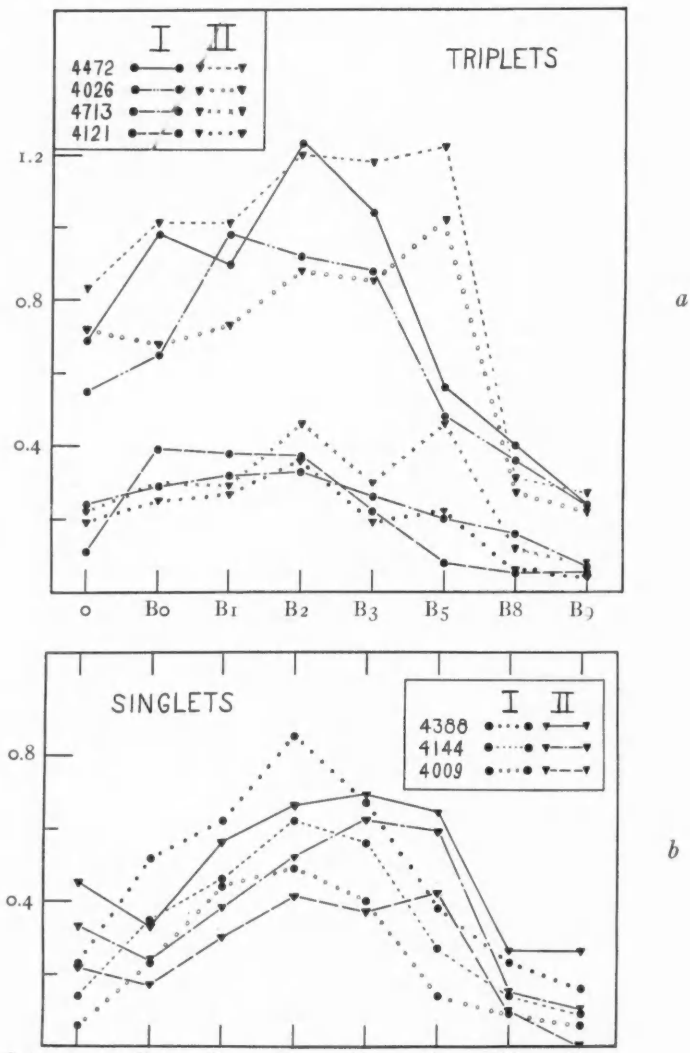
The behavior with changing Harvard spectral type of the helium lines in the sharp-line (class I) and wide-line (class II) stars is shown in Figure 10. There appears to be no sensible difference between singlets and triplets. All curves for class I appear normal (cf. Fig. 4), but those for class II are quite definitely different, showing broad maxima more than a whole subtype later than the sharp-line stars. This suggests that in the Harvard system the rotation stars are systematically classified too late, if we take helium intensities for the criterion of spectral type. It has already been proved that, if hydrogen intensities are used as the criterion, there is no such effect. The shift of maximum is still definitely present, though of course diminished, if the peculiar B5 wide-line star 27 Canis Majoris is entirely omitted from the discussion. The plate measured for this star, however, is well guided and of good quality. For the stronger singlet lines, indeed, one may disregard the points for class B5 entirely, and there still remains a very definite shift of the maximum.

When Victoria subtypes are once more substituted for Harvard, as was done for the hydrogen intensities, retaining the division into classes I and II as before, the results are as shown in Figure 11. The non-rotating stars again show maximum intensities at B2, while for the class II stars the maximum is definitely shifted to B3.

Not enough of the seventy stars measured for this investigation have been measured by the workers at Mount Wilson to attempt a comparison on the basis of their classification.

[24] Miss Payne *et al.* found the same effect in the stars of the Scorpio-Centaurus cluster (*Harvard Circ.*, No. 365, 1931).

[25] Miss Payne *et al.* believed such an enhancement of *He* I in the giants to be present in the Scorpio-Centaurus cluster (*ibid.*).



No. of stars	o	Bo	B1	B2	B3	B5	B8	B9
Class I:	6	5	1	3	6	9	6	4
Class II:	4	4	4	2	8	2	4	2

FIG. 10.—Behavior of neutral helium intensities with Harvard spectral type, in sharp-line (class I) and wide-line (class II) stars. Ordinates are angstroms total absorption.

V. BEHAVIOR OF OTHER ELEMENTS

The behavior with Harvard spectral type of such other elements as were measured in an appreciable number of stars (i.e., elements besides *H* and *He* I) is shown in Figure 12. Each point represents the mean for all stars studied in that spectral type; absences were counted as zero intensities in taking these means. Only when a line had

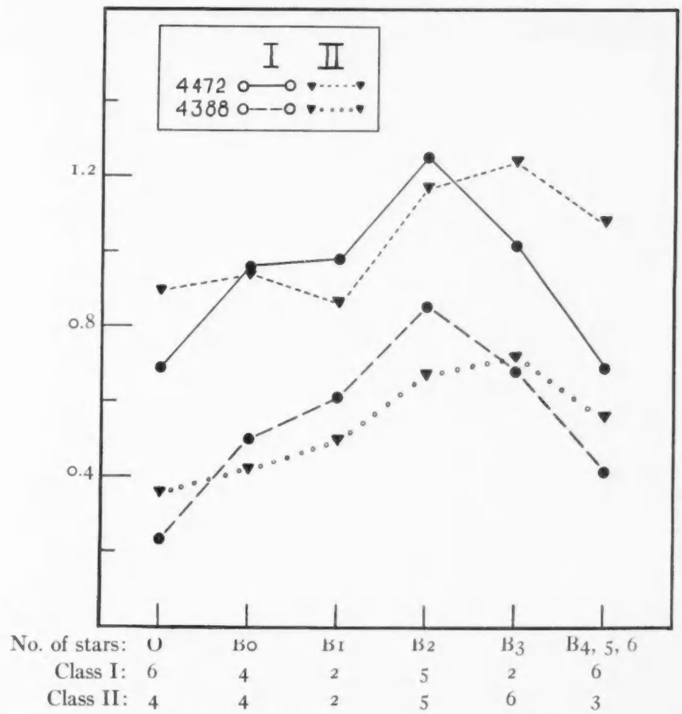


FIG. 11.—Behavior of strong helium lines with Victoria spectral type, in sharp-line (class I) and wide-line (class II) stars. Intensities are in angstroms total absorption.

been measured on the majority of plates of one Harvard subtype was its absence determined or its intensity estimated visually in the rest of the group and the mean intensity plotted in Figure 12.

The agreement with previous results, based in great part on estimated data, is excellent; there are only a few minor exceptions. Comparing with Miss Payne's data,[26] we find substantiated the excellent

[26] *M.N.*, **92**, 368, 1932; see also *Harvard Circ.*, No. 256, 1924. For special reference to O stars, see H. H. Plaskett, *Pub. Dom. Ap. Obs.*, **1**, No. 30, 1922.

correlation which she shows between spectral type of maximum intensity and ionization potential for the various atoms and ions; there are, however, disagreements of one Harvard subclass as to the position of maximum for *N* III, *Si* IV, and *C* II. The first two of these may be due to the fact that I have considered all of the O stars measured as one group. The maximum found from my measures at type B8 for *Si* II may be spurious, owing to luminosity effects;

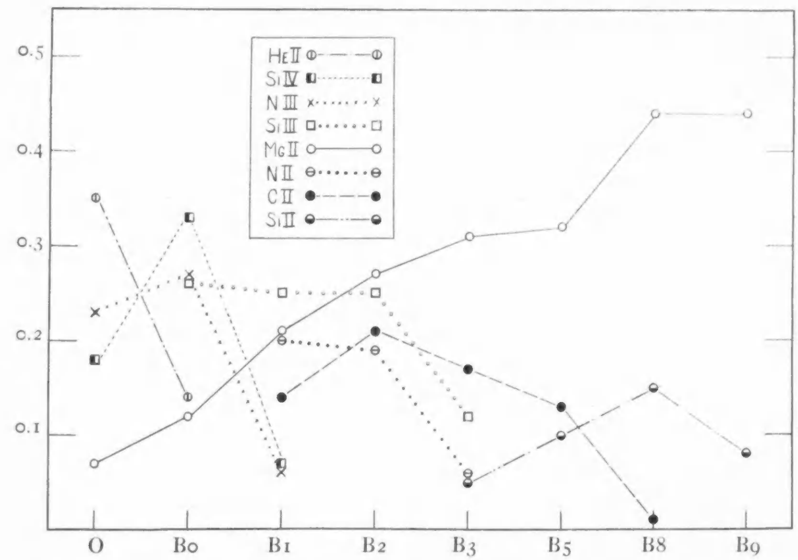


FIG. 12.—Behavior of line intensities of various elements (expressed in angstroms total absorption) with Harvard spectral type. Points shown are means for the following lines: *He* II, λλ 4200, 4542, and 4686; *Si* IV, λλ 4089, 4116; *N* III, λ 4097; *Si* III, λλ 4552, 4567, 4574; *Mg* II, λ 4481; *N* II, λ 3995; *C* II, λ 4267; *Si* II, λλ 4128, 4131.

there are three giants, in which these lines are enhanced, in the B8 group, and none in B9. Measures of spectra of types A0 and A2 will be needed to settle this point.

One is impressed, upon considering the implications of Figure 12, with the large change in spectrum corresponding to an interval of one subtype in the range earlier than B2, compared with the later classes. Compare, for instance, the breadth of the maximum for *C* II with the sharpness of the peaks for *Si* IV and *N* III.

A study of these elements, with the stars divided into giant and

dwarf groups, yields several interesting results, shown in Figure 13. λ 4481 of *Mg* II shows no appreciable effect until B2, agreeing with Morgan;[27] in the giants of types B8 and B5 it is about 40 per cent

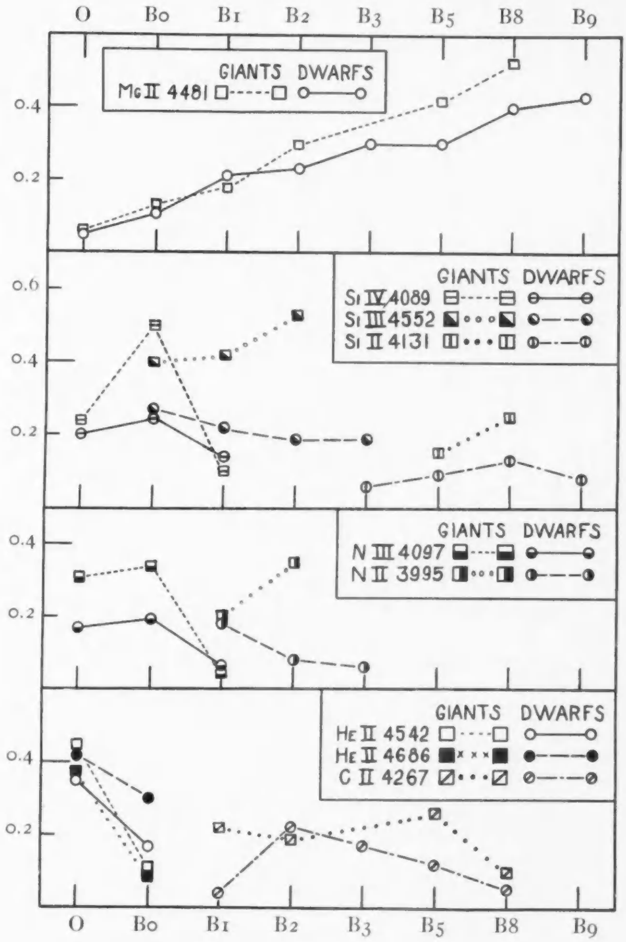


FIG. 13.—Behavior with spectral type, in giant and dwarf groups of stars, of line intensities of (*a*) magnesium, (*b*) silicon, (*c*) nitrogen, and (*d*) ionized helium and carbon.

stronger than in the dwarfs.[28] The ratio λ 4481 : λ 4472, often used for a criterion of spectral type, will therefore show an absolute-magnitude effect, since one line or the other is sensitive to absolute mag-

[27] *Ap. J.*, **77**, 291, 1933.

[28] This strengthening was noticed by Struve (*ibid.*, **78**, 82, 1933).

nitude in every subdivision of the B stars. Every one of the measured lines of silicon and nitrogen in various stages of ionization shows approximately twice the maximum strength in giants that it shows in dwarfs. *C* II also is enhanced in the B5 and B8 giants.

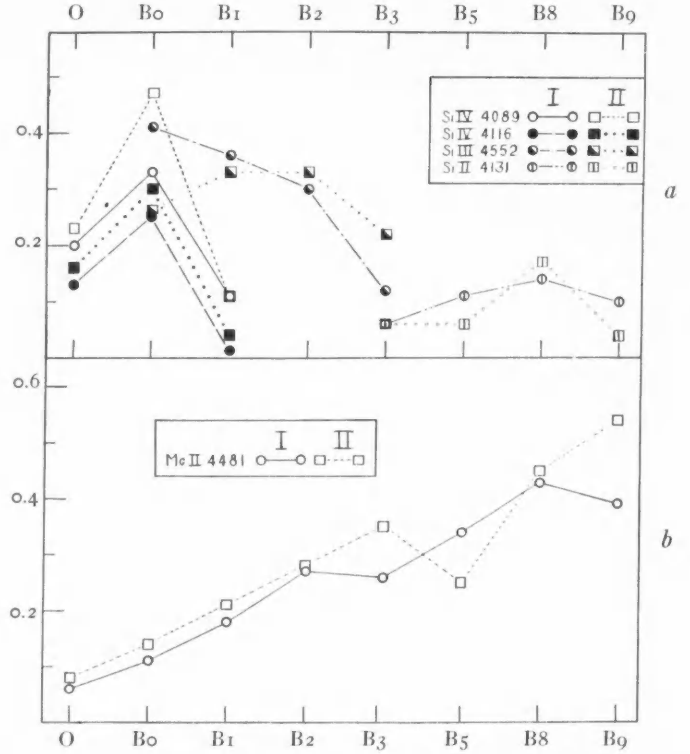


FIG. 14.—Behavior with Harvard spectral type of line intensities of (*a*) silicon and (*b*) magnesium, in stars with sharp lines (class I) and with wide lines (class II). Ordinates are angstroms total absorption.

Thus, the spectra of *He* I (in the types earlier than B3) and of hydrogen remain the only ones in which the lines are consistently more intense in dwarfs than in giant stars.

The behavior of ionized helium should be mentioned in this connection. Referring to Figure 13 (*d*), we see that the mean intensities of λ 4686 and λ 4542 (members of different series) are not greatly different in the giants and dwarfs of type O, but at B0 they are

stronger in the dwarfs. This effect is more marked in λ 4686, with the result that this line is three times as strong in B0 dwarfs as in giants, while λ 4542 is only 50 per cent strengthened in the dwarf group. The noteworthy feature of this effect is that the B0 stars in Orion, as mentioned in section IV, show the same thing; that is, λ 4686 is more intense in the fainter and presumably less luminous of those stars, while λ 4542 shows no definite correlation.

Upon investigating the effects of rotation (Fig. 14), we find that nearly all of these elements show no systematic effect (e.g., *Si* II, *Mg* II); λ 4089 of *Si* IV, however, is apparently definitely stronger in the rotating stars (class II) at B0. No obvious explanation suggests itself; λ 4116, also due to *Si* IV, shows the effect only to an extent well within the errors of measurement. *Si* III also seems to be stronger in the class II stars at B0, though not in the later types.

I wish to express my sincere thanks to Dr. Otto Struve and to Dr. W. W. Morgan for suggesting this problem and for their frequent and helpful advice, and to Dr. H. Rosenberg for his kindness in allowing me to use his photometer.

YERKES OBSERVATORY
WILLIAMS BAY, WISCONSIN
December 28, 1935

THE CLASSIFICATION AND LUMINOSITY OF THE n AND s STARS OF TYPE A

J. A. HYNEK

ABSTRACT

Data are compiled concerning the luminosity, the color temperature, and the spectral classification of 409 of the Mount Wilson n and s stars of type A. It is shown that 255 n stars show a strong tendency to cluster closely about the spectroscopic absolute magnitude +1.9. No systematic error is found in the determination of the luminosities of these stars spectroscopically and from trigonometric parallaxes. It is shown that observed differences in luminosity between the n and s stars of a given spectral class are probably due to the preferential luminosity of the n stars rather than to an absolute-magnitude effect caused by rapid axial rotation.

It is shown that the differences in color observed when the n and s stars are arranged according to Mount Wilson classes disappear when they are arranged according to Harvard spectral classes. This is strong negative observational evidence for a rotational absolute-magnitude effect. The consistency of the Harvard classification of Mount Wilson n and s stars is confirmed by a quantitative examination of the intensities of the most prominent of the usual criteria of spectral classification of A stars.

The effect of rapid axial rotation upon the number of atoms active in the production of a few prominent lines of an A-type spectrum is estimated, and it is shown that the effect is so small that it would be difficult to detect it observationally, except perhaps in individual rapidly rotating giants.

Recent papers by Struve[1] and by Miss Williams[2] have considered certain aspects of the physical significance of the n and s stars.[3] It was pointed out that, in general, the n stars are less luminous than the s stars of the same spectral class. Three possible causes of this phenomenon were given: (*a*) there may be a systematic error in the classification of the n stars, (*b*) the more luminous stars rotate more slowly than the less luminous ones, (*c*) the decrease in surface gravity due to the rapid rotation of the less luminous star makes it appear to belong to an earlier spectral class than it would if it had no rotation. Of these three possibilities, Miss Williams, from the

[1] *Pop. Astr.*, **43**, 496, 1935.

[2] *Ap. J.*, **82**, 432, 1935.

[3] There exists some confusion in the use of this notation. The suffixes are mutually exclusive (Int. Astr. Un., 1922); "n" denotes a spectrum having all lines diffuse, "s" is used when the lines are uniformly sharp but when "c" characteristics are not present. The two are used simultaneously by some writers to denote intermediate stages between sharp- and very diffuse-line stars, while certain stars have been classed as "n" by some observers and "s" by others. The latter difference probably arises when the hydrogen lines are markedly broadened by the Stark effect, while the metallic lines remain sharp.

study of the spectral classes, luminosities, and color temperatures of 54 stars taken from the catalogue of Adams and Joy,[4] concluded that (*c*) was the most probable cause. Struve and Morgan, on the other hand, have expressed the opinion (private communication) that the effect found by Miss Williams is probably caused by (*a*) and that a careful study of (*a*) and (*b*) should precede any attempt to evaluate (*c*). A study of 409 n and s stars listed in the recent catalogue of spectroscopic parallaxes by Adams and his collaborators[5] does not confirm the conclusion of Miss Williams, and indicates strongly that both causes (*a*) and (*b*) are present but that (*c*) is practically absent.

THE LUMINOSITIES OF THE n AND s STARS

Table I*a* gives the distribution of the n and s stars with respect to the Mount Wilson spectral classes; Table I*b* gives a similar distribution with respect to the Harvard spectral classes, and Table I*c* is Miss Williams' original table,[6] given for comparison. The first two tables contain, in successive columns, the spectral class of the s stars, the mean absolute magnitude, the mean value of the color equivalent c_2/T for all stars for which it was available, the weight of the color determinations, which is taken equal to the number of determinations available, the number of stars, and the total range in absolute magnitude within each subclass. The last six columns give the same data for the n stars. Miss Williams' table is similar in content. In the recent Mount Wilson catalogue of spectroscopic parallaxes, stars earlier than A0 are not included and the suffixes n and s are not applied to stars later than A9, while all the A stars listed have either the suffix n or the suffix s. Table I*b* has fewer stars than Table I*a* because many of the Mount Wilson A stars are listed as composite by Harvard, a few do not have corresponding Harvard classes, and some are either earlier or later than A0 or F0, respectively, in the Harvard notation. These stars were not included in Table I*b*.

A glance at Tables I*a* and I*b*, columns 2 and 8, shows at once that in the redetermination of luminosities at Mount Wilson the familiar correlation of the n stars with luminosity, which for some time was

[4] *Mt. W. Contr.*, No. 244, 1922. [5] *Ap. J.*, **81**, 187, 1935. [6] *Op. cit.*

TABLE I*a*

DISTRIBUTION OF n AND s STARS AMONG MOUNT WILSON SPECTRAL CLASSES

Mount Wilson Class	Mean M	M.V. c_2/T	Wt.	No. of Stars	Total Range in M	Mount Wilson Class	Mean M	M.V. c_2/T	Wt.	No. of Stars	Total Range in M
A0s	....	....	....	0	....	A0n	+2.2	....	....	2	0.1
A1s	+1.0	1.49	1	3	0.6	A1n	2.0	1.60	7	17	0.8
A2s	1.0	1.58	10	16	1.8	A2n	2.0	1.69	15	27	0.8
A3s	1.1	1.67	12	16	1.8	A3n	1.9	1.84	12	27	0.8
A4s	1.6	1.62	9	19	4.7	A4n	1.9	1.92	13	31	0.8
A5s	1.4	1.79	4	11	4.8	A5n	1.9	2.03	13	36	0.7
A6s	1.4	1.79	5	14	1.2	A6n	1.8	2.16	13	43	1.0
A7s	1.6	1.96	7	25	4.4	A7n	1.8	2.20	6	26	1.3
A8s	2.1	2.03	3	26	5.7	A8n	2.0	2.12	4	21	1.3
A9s	+2.3	2.23	2	26	4.0	A9n	+2.0	1.69	2	23	1.8
F0s	....	....	....	0	....	F0n	....	....	....	0	....
Mean	+1.5	1.80	....	156	3.2	Mean	+1.9	1.92	....	253	0.9

TABLE I*b*

DISTRIBUTION OF n AND s STARS AMONG HARVARD SPECTRAL CLASSES

Harvard Class	Mean M	M.V. c_2/T	Wt.	No. of Stars	Total Range in M	Harvard Class	Mean M	M.V. c_2/T	Wt.	No. of Stars	Total Range in M
A0 (s)	+1.3	1.56	16	29	2.2	A0 (n)	+2.1	1.53	7	33	1.0
A2 (s)	1.2	1.64	19	37	2.1	A2 (n)	1.9	1.68	9	32	0.9
A3 (s)	1.7	1.88	6	22	2.3	A3 (n)	1.9	1.74	9	27	1.1
A5 (s)	1.7	1.92	4	11	3.0	A5 (n)	1.9	1.88	20	37	0.9
F0 (s)	+1.9	2.02	10	30	4.8	F0 (n)	+1.9	2.13	34	90	1.3
Mean	+1.6	1.80	....	129	2.9	Mean	+1.9	1.79	....	219	1.0

TABLE I*c*

MISS WILLIAMS' TABLE OF n AND s STARS

Adams and Joy Class	M	M.V. c_2/T	No. of Stars	Adams and Joy Class	M	M.V. c_2/T	No. of Stars
B9s	−0.2	1.56	2	B9n	+0.5	1.64	4
A0s	+0.2	1.58	1	A0n	0.9	1.67	8
A1s	0.6	1.57	3	A1n	1.3	1.67	9
A2s	0.9	1.62	7	A2n	1.7	1.82	6
A3s	1.2	1.72	3	A3n	2.0	1.89	2
A4s	1.6	1.73	2	A4n	2.2	2.08	3
A5s	+1.8	1.83	2	A5n	+2.3	2.13	2
Mean	+0.7	1.66	20	Mean	+1.6	1.84	34

used as a means of determining the spectroscopic parallaxes of A-type stars,[7] has disappeared. The luminosity of the n stars now appears to be practically constant regardless of the spectral subclass. Thus the very effect for which an explanation was sought, the fairly uniform difference in luminosity between the n and s stars, has disappeared. The hypothesis of a rotational absolute-magnitude effect could now only be applied to the early A stars and would fail completely for the late A's, in which there is even some evidence for a difference of luminosity in the opposite sense from that in the early A's, unless we assume that the rotation of the late A stars is considerably less than that of the early A stars. The mean luminosity for 255 Mount Wilson An stars is +1.9, the mean deviation being but 0.25 mag. This is shown graphically in Figure 1, in which the luminosities of the Mount Wilson n and s stars are plotted against spectral class. The last column in Table I*a* shows that the mean total range in luminosity within each subclass for the s stars is 3.2 mag., while for the n stars it is only 0.9 mag. Figures 1 and 2 show that the extreme range in luminosity for all the s stars is almost 7 mag., while for the n stars it is less than 2 mag. It seems very probable, therefore, that what luminosity difference is observed between the n and s stars of early A type can be explained by the fact that the normal sharp-line stars decrease in luminosity in the normal fashion as later spectral types are approached, but that the n stars cluster closely about a preferred absolute magnitude of +1.9.

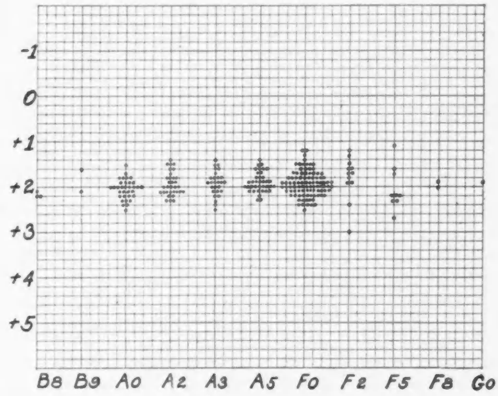


FIG. 1.—The luminosities of the Mount Wilson An stars plotted against Harvard spectral classes as abscissae. The uniform clustering for all subclasses is clearly shown.

There is some evidence that the same is true for B and O stars.

[7] *Mt. W. Contr.*, No. 244, 1922.

For these classes neither good trigonometric nor spectroscopic parallaxes are available. The Mount Wilson catalogue of spectroscopic parallaxes lists only five rotating B stars, but their mean spectroscopic absolute magnitude is +2.0, the variation being less than half a magnitude. An examination of Struve's table of B and O stars[8] shows that giant and supergiant stars exhibit the least rotation, the maximum rotation occurring in the intermediate and dwarf stars, and more often in the dwarf stars. This places the preferential luminosity for these stars at about +1.0.

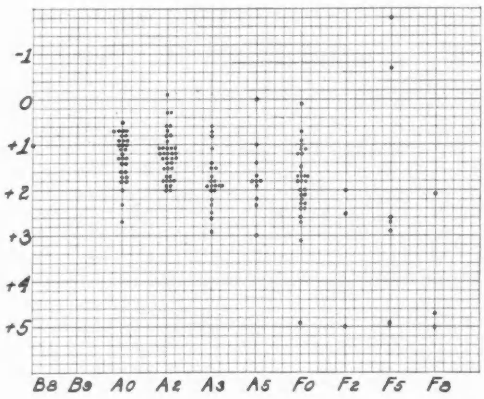


FIG. 2.—The luminosities of the Mount Wilson As stars plotted against Harvard spectral classes as abscissae. The normal Russell diagram distribution is followed.

It must be recalled that only luminosities derived from spectroscopic parallaxes were used in the foregoing. To test whether the remarkable grouping of n stars about a given absolute magnitude might be due to a systematic difference in the assignment of spectroscopic parallaxes, owing to the difficulty of estimating the intensities of the "dish-shaped" lines in the n stars, the luminosities of these stars derived from trigonometric parallaxes were examined. Trigonometric parallaxes are available for only 64 out of the 255 stars. The mean luminosity of these stars derived from trigonometric parallaxes is +1.6 and the scatter is considerably greater than for the spectroscopic parallaxes. The scatter is easily accounted for, however, by the probable error of the trigonometric parallaxes. The mean difference between the absolute magnitudes determined trigonometrically and those determined spectroscopically $(M_t - M_s)$ for the 64 stars is 1.1 mag. The average probable error of the absolute magnitude associated with the probable error of the trigonometric parallaxes (ΔM_t) is also 1.1 mag. The scatter is therefore equal to

[8] *Ap. J.*, **74**, 225, 1931.

the probable error, and we can conclude that no large systematic error enters into the determination of the spectroscopic parallaxes of rapidly rotating stars. Further confirmation of this is given by a study of the ratio $(M_t - M_s/\Delta M_t)$; i.e., the difference in absolute magnitudes determined trigonometrically and spectroscopically, in terms of the probable error of the absolute magnitude associated with the probable error of the trigonometric parallax, formed for some seventeen hundred stars of the Mount Wilson catalogue for which trigonometric parallaxes were available.[9] The mean value of the ratio is 0.8, and its similarity to that for the rotating stars indicates that the spectroscopic parallaxes of the n stars are not systematically different from those of the other stars. In view of the foregoing, we are led to the conclusion that the preferential absolute magnitude of the n stars is real, and that the suggested absolute-magnitude effect due to rapid axial rotation does not appear to be present in any marked degree since there is no longer a constant systematic shift in luminosity between n and s stars of a given spectral class.

THE COLORS AND THE CLASSIFICATION OF THE n AND s STARS

Both Table I*a* and Table I*c* show the systematic difference in color for the n and s stars of each spectral subclass, which led Miss Williams to conclude that the then observed systematic shift in luminosity of the n stars with respect to the s stars was due to a spurious absolute-magnitude effect resulting from rapid rotation. For, if a star of a given spectral class be given a rapid rotation, the higher level of ionization resulting from the lowered surface gravity would lead to an earlier spectral class, while the color of the star would remain the same.

An examination of Table I*b*, however, shows that when Harvard spectral classes are used, the difference in color also disappears. The n and s stars of each subclass have the same mean color equivalents. The weights assigned represent the number of observations available. The colors have been obtained from the catalogues of Becker and Bottlinger,[10] of Hertzsprung,[11] and of Miss Williams.[12] The

[9] Unpublished material.

[10] *Veröff. Berlin-Babelsberg Obs.*, **10**, No. 3, 1933.

[11] *Ann. Leiden Obs.*, **14**, Part I, 1922.

[12] *Harvard Circ.*, **248**, 1929.

color indices given by Becker and Bottlinger were plotted against the color equivalents by Hertzsprung, and the resulting curve was used as a reduction-curve to make the results of the two catalogues comparable. If the Harvard classification of the n stars is consistent with respect to the usual criteria of classification, then this is added evidence that axial rotation has no marked effect on the spectral class, for the latter would demand a difference in color between the n and s stars of a given spectral class.

TABLE II

Line	Harvard Classes			Mount Wilson Classes		
	A0–A2(s); 9 Stars	A3–A5(s); 2 Stars	F0(s); 6 Stars	A0s, A1s, A2s; 5 Stars	A3s, A4s, A5s; 6 Stars	A6s, A7s, A8s; 3 Stars
	A0–A2(n); 6 Stars	A3–A5(n); 6 Stars	F0(n); 7 Stars	A0n, A1n, A2n; 8 Stars	A3n, A4n, A5n; 7 Stars	A6n, A7n, A8n; 3 Stars
Ca II K	12 12	26 28	54 47	8 16	21 36	30 48
$H\gamma$	91 92	100 92	62 62	72 90	95 81	91 57
Fe I 4045	(14) (6)	(9) 8	16 16	 8	9 11	14 17
Ca I 4226	(6) (6)	10 12	21 20	6 9	8 15	14 17
Sr II 4078	(17) 6	(7) 8	24 19	7 9	11 12	22 21

The consistency of the Harvard system of classification with respect to the sharp- and diffuse-line F0 stars has already been indicated[13] by the fact that the intensities of the K line of ionized calcium and $H\delta$ are the same for both the sharp- and the diffuse-line F0 stars. Reference to the spectrophotometric data compiled by Miss Williams[14] gives further evidence of the validity of the Harvard classification of n and s stars. Table II contains her data arranged in the first three columns in terms of the Harvard classes, and in the last three columns in terms of the Mount Wilson classes. The

[13] *Ap. J.*, **82**, 338, 1935.

[14] *Op. cit.*

top number in each row refers to the intensities of the selected lines in the sharp-line stars, and the lower number to the intensities in the diffuse-line stars of the same spectral class. Total absorptions are given for K and $H\gamma$, while line depths in hundredths of a magnitude are listed for *Fe* I 4045, *Ca* I 4226, and *Sr* II 4078.

It is apparent at once that the Harvard classification of the n stars is more consistent with respect to the normal stars of the same class than is the Mount Wilson classification, when these criteria are employed. The two most prominent features in the A-type spectrum, the K line of calcium and the hydrogen lines, as criteria, indicate that, in general, the n stars are classified too early by Mount Wilson. It is well known that the K line grows steadily stronger in successive subdivisions of the A spectrum, while the hydrogen lines have a maximum at about A2. Table II shows that the total absorption of the K line is systematically greater for the n stars and that the hydrogen lines pass through their maximum much earlier in the n stars than in the s stars. These differences would disappear if the n stars were, on the whole, classified about 0.4 of a spectral class later.

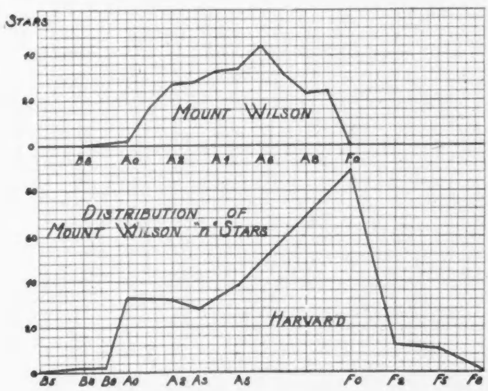


FIG. 3.—The Mount Wilson An stars arranged according to the Mount Wilson and Harvard spectral classes. The left-hand portion of the graph is of no significance since stars earlier than A are not included. The rapid falling-off at the right, however, is real.

Figure 3 gives the frequency distribution of the 255 n stars among the Mount Wilson and the Harvard spectral classes. Since stars earlier than A are not included, the left-hand portion of the graph is of no significance. The peaking of the n stars at Mount Wilson class A6n and at Harvard F0 is of considerable significance in view of the foregoing. The rapid diminution in number of n stars as later spectral types are approached has previously been pointed out.[15]

[15] *Ibid.; Ap. J.*, **79**, 357, 1934.

The neutral lines of iron and of calcium, and the ionized line of strontium are very weak in the A stars, but Table II indicates somewhat that these also show the Mount Wilson n stars to be classified too early according to these standards. The parentheses indicate that only one measure has been available. In spite of the small number of stars used, the results are consistent, the small scatter confirming the accuracy of Miss Williams' observations.

We can interpret these results as a slight shift in the Mount Wilson classifications of the n and s stars. This systematic shift has previously been discussed by Lindblad,[16] Miss Fairfield,[17] and others. There are, however, many individual cases in which the Mount Wilson classification of the n stars is later than the Harvard classification. The mean color equivalent is available for only 12 stars of this type. The mean Harvard spectral class is A1.8, the mean Mount Wilson class is A4.2, and the mean color equivalent is 1.64, which corresponds to that of an A1.5 star.[18] Further, for those n stars which both Mount Wilson and Harvard classify as A-type stars, the mean color equivalent, 1.76, is in excellent agreement with the mean spectral class in both systems, A3. For those n stars, however, which are A stars in the Mount Wilson system (mean spectral class, A6) and F stars in the Harvard system (mean spectral class, F0), the mean color equivalent for 18 such stars is 2.15, the color of a typical F0 star.

The two systems of classification differ somewhat also for the sharp-line stars. Tables I*a* and I*b* show that the Mount Wilson As stars embrace a greater range in temperature and luminosity than do the Harvard A stars. From A1s to A9s in the Mount Wilson system the color equivalent ranges from 1.49 to 2.23, and the luminosity from +1.0 to +2.3, while from Harvard A0 to F0 the color range is only from 1.56 to 2.02, and in luminosity from +1.3 to +1.9.

THE ROTATIONAL ABSOLUTE-MAGNITUDE EFFECT

The absence of the color difference and of a uniform shift in luminosity between the n and s stars of a given spectral class indicates that the expected effect of rotation on the spectrum does not

[16] *Ap. J.*, **59**, 305, 1924.

[17] *Harvard Circ.*, **264**, 1924.

[18] *Ann. Leiden Obs.*, **14**, Part I, 15, 1922.

occur in any marked degree. The absence of such an effect is of theoretical interest. The studies of Elvey[19] and of Miss Westgate[20] show the mean linear equatorial velocity of late A-type stars and early F stars to be very close to 100 km/sec and the maximum velocity to be 250 km/sec. Adopting a temperature of 7200° C ($z'=0.70$)[21] and the mean observed luminosity of the n stars, $+1.9$, the hypothetical radius corresponding to these values is $2.6\times\odot$, or 1.83×10^{11} cm.[22] The centrifugal force at the equator will reduce the surface gravity by 0.55×10^3 cm/sec^2 for the mean case, and by 3.42×10^3 cm/sec^2 in the extreme case of 250 km/sec. Since the maximum effects of the centrifugal force are felt only at the equator, it is necessary to multiply this value by the mean value of $\cos^2\phi$, $\frac{2}{3}$, where ϕ is the latitude, to arrive at the mean reduction in surface gravity. These values are then 0.36×10^3 cm/sec^2 and 2.28×10^3 cm/sec^2 for the mean and extreme cases, respectively. Russell's formula for g for dwarf stars, $\log g/g^0=-0.65+0.28z$,[23] where g^0 is the surface gravity of the sun and $z=11{,}600/T$, gives a value of 1.7×10^4 for the surface gravity of an Fo dwarf ($T=7200$). Thus:

$$g_d-g_d'=0.04\times10^4 \text{ (for the mean case) },$$

$$g_d-g_d'=0.23\times10^4 \text{ (for the extreme case) },$$

where the subscript d refers to a dwarf star and the prime to a rotating star. Russell has used $g_d-g_G=1.29\times10^4$ cm/sec^2 where the subscript G refers to giants, in computing the curves[24] for the numbers of atoms active in the production of certain lines in giants and dwarfs of type Fo.

In the mean, therefore, the effect of rotation is practically negligible. Even in the case of maximum velocity the effects are small, the change in mean surface gravity being scarcely one-fifth of that between dwarfs and giants. Table III gives the estimated changes in the number of atoms effective in producing a line on passing from a non-rotating star to one of equatorial velocity 250 km/sec. Column 1 gives the line, column 2 the computed difference in loga-

[19] *Ap. J.*, **71**, 221, 1930. [20] *Ibid.*, **78**, 46, 1933. [21] *Ibid.*, p. 239.

[22] Russell, Dugan, and Stewart, *Astronomy*, **2**, 738.

[23] *Ap. J.*, **78**, 239, 1933. [24] *Ibid.*, pp. 289, 290.

rithms of the number of atoms effective in producing the line between giants and dwarfs, taken from Russell's published graphs,[25] for $z' = 0.7$ (Fo). Column 3 lists the ratio of effective atoms in a non-rotating dwarf star to those in a rapidly rotating star of class Fo, and column 4 gives the square root of this ratio, since the total absorption of strong lines is generally taken to vary as the square root of the number of atoms concerned in their production.

It is seen that even in the extreme case of an equatorial velocity of 250 km/sec the change in surface gravity is such as to change the total absorption of *Ca* I 4226, the most sensitive line considered, by less than 5 per cent. The other lines would be affected in lesser degree.

TABLE III

Line	log L_d/L_g	$L_d/L_{d'}$	$\sqrt{L_d/L_{d'}}$
Ca II K	$<+0.03$	1.01	1.01
Ca I 4226	$+\ .30$	1.10	1.05
$H\gamma$	$-\ .10$	0.96	0.98
Fe I 4383	$<+0.20$	1.09	1.05

It would seem, therefore, in the particular case assumed above, that the rotational absolute-magnitude effect should be very small. It must be remembered that this result depends on the assumption of a hypothetical radius ($2.6\odot$) corresponding to an absolute magnitude of $+1.9$. Since, however, the centrifugal force varies inversely as R (for a given surface velocity), while surface gravity varies inversely as R^2, a small change in the centrifugal force in a rotating supergiant star would be highly important and would cause a marked absolute-magnitude effect and even a mechanical instability of the star, as has been suggested by Struve.[26] No rapidly rotating supergiants are known among the A stars—perhaps for that very reason—and statistically the effect of rotation in the intermediate stars of class A should be small.

PERKINS OBSERVATORY
DELAWARE, OHIO
January 1936

[25] *Ibid.*

[26] *Ibid.*, **72**, 1, 1930.

THE Be SPECTRUM VARIABLE, χ OPHIUCHI

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ABSTRACT

χ Ophiuchi shows correlated changes of hydrogen velocity and emission ratio, like other Be spectrum variables. A period of about ten years seems probable. The iron emission shows a similar period, but the helium absorption is out of phase and is more irregular. The helium singlets, λ 4144 and λ 4388, give a mean velocity of 22 km/sec greater than the helium triplets, λ 4121 and λ 4472.

χ Ophiuchi[1] belongs to spectral class Be, which is characterized by emission lines of hydrogen superimposed upon diffuse absorption lines. The spectrum of this star has unusually strong emission lines, which usually appear single on the Ann Arbor spectrograms. Variation in the relative intensity of the violet and red edges of the lines was found by McLaughlin.[2]

Changes in the emission ratio in other Be stars have been found to be accompanied by changes in radial velocity. This star was found to have a variable velocity, from two spectrograms in 1896 and one in 1902, taken at the Lick Observatory.[3] Measurements of the central absorption of $H\gamma$ on these three-prism plates gave -11 km/sec for the first two and $+21$ km/sec for the third. In 1925, four one-prism plates obtained at the Lick Observatory[4] gave a mean velocity of -11 km/sec. However, this value depends on the lines of several elements in addition to hydrogen.

The object of this investigation is to determine radial velocities from the lines of hydrogen, helium, and ionized iron, on an extensive series of spectrograms, and to correlate the velocities with the changes of the emission lines.

THE OBSERVATIONS

This study is based upon 113 one-prism spectrograms of χ Ophiuchi taken at the Observatory of the University of Michigan. Five plates were obtained in 1913 and the rest during the interval

[1] α $16^{h}21^{m}.2$, δ $-18°14'$, mag. 4.9, spectrum B3e.

[2] *Pub. U. of Michigan Obs.*, **4**, 184, 1932.

[3] *Lick Obs. Bull.*, **5**, 175, 1910.

[4] *Pub. Lick Obs.*, **16**, 240, 1928.

1923–1934, inclusive. The southern declination of the star limits the observing season, so that most of the observations were made during the four months April–July, inclusive.

The hydrogen lines $H\beta$, $H\gamma$, and $H\delta$ were measured on almost all the plates. Three settings were made on each line, one on each edge and one bisecting it. The velocity was obtained by averaging the bisection setting with the mean of the edge settings.

Nine lines of ionized iron are definitely present as emission, but most of them are very poor, and only four have been included in the results. These are λλ 4233, 4549, 4556, and 4584. Similarly, of the seven absorption lines of helium which were measured, only four have been used. These are λλ 4121, 4144, 4388, and 4472. Because of atmospheric extinction of the violet light, resulting from the low altitude of the star, the helium line λ 4026 was too poorly exposed on most of the plates. The line K of ionized calcium was measured on about one-third of the plates. The values of hydrogen emission ratio are those found by McLaughlin.[5]

TABLE I

NORMAL PLACES OF HYDROGEN VELOCITIES

Epoch	JD 242+*	$H\beta$	$H\gamma$	$H\delta$	Weighted Mean	No. of Plates
1913.4	9914*	+2	+1	+7	+3	5
1923.3	3534	+1	+4	+12	+5	8
1924.4	3924	+3	+4	+5	+4	11
1925.4	4302	−4	−2	0	−2	7
1926.4	4668	−8	−7	−9	−8	8
1928.6	5471	−25	−21	−13	−20	1
1929.5	5784	−9	−6	−1	−6	21
1930.5	6142	−6	−6	+2	−4	15
1931.4	6479	−5	−1	−4	−3	7
1932.4	6866	0	0	0	0	7
1933.4	7235	+3	+7	+10	+7	17
1934.5	7615	+3	+6	+9	+6	6

* First three figures are 241 for first entry.

RESULTS OF THE OBSERVATIONS

Tables I–IV give the annual means of the individual lines and of the weighted means. $H\gamma$ was given a weight of 2, and $H\beta$ and $H\delta$

[5] *Op. cit.*, p. 185.

each a weight of 1. Of the ionized iron lines, λ 4233 has a weight 2 and the others each 1. The helium lines have been divided into two

TABLE II

NORMAL PLACES OF IRON VELOCITIES

JD 242+*	4233	4549	4556	4584	Weighted Mean	No. of Plates
9914*........	+43	+ 7	−10	+ 7	+15	5
3534........	33	+17	+11	+ 2	+19	8
3924........	22	+ 3	+25	+13	+16	10
4302........	9	−21	−10	−13	− 4	6
4668........	8	− 9	−15	− 8	− 2	6
5471........	15	− 5	+13	−35	+ 1	1
5784........	3	− 4	+18	− 9	+ 2	20
6142........	6	+ 2	+16	+ 8	+ 8	13
6479........	20	− 9	+15	−17	+ 5	7
6866........	38	−14	+20	+ 7	+17	7
7235........	16	− 4	+ 9	− 3	+ 7	15
7615........	+33	+14	+25	+ 4	+21	6

* First three figures are 241 for first entry.

TABLE III

NORMAL PLACES OF HELIUM VELOCITIES

JD 242+*	TRIPLETS			SINGLETS			WEIGHTED MEAN OF 4 LINES	NO. OF PLATES
	4121	4472	Weighted Mean	4144	4388	Weighted Mean		
9914*............	−39	− 8	−11	+11	+20	+17	+ 3	5
3534............	−23	14	16	− 4	+ 5	+ 2	− 7	8
3924............	−10	22	18	+ 3	+ 2	+ 2	− 8	11
4302............	−15	26	22	+ 9	+ 1	+ 3	− 9	6
4668............	−25	18	19	− 9	− 7	− 8	−14	6
5471............	− 2	17	12	+40	−26	− 4	− 8	1
5784............	+11	10	4	− 1	+ 6	+ 4	0	19
6142............	−10	13	12	+14	+22	+20	+ 4	15
6479............	+ 2	12	8	+ 8	+ 2	+ 4	− 2	7
6866............	−25	11	14	+16	+11	+13	− 1	7
7235............	−17	26	23	+ 6	+ 6	+ 6	− 8	15
7615............	− 6	−30	−23	+29	+13	+17	− 3	6

* First three figures are 241 for first entry.

groups, triplets and singlets. Of the triplets, λ 4472 has twice the weight of λ 4121; and of the singlets, λ 4388 has twice the weight of λ 4144. Table V gives the widths in angstroms and the emission ratios of the hydrogen lines.

As a check on some of the measurements, it is worth while to include a summary of the results obtained by two other measurers. In 1924, P. A. Smith measured 6 of the 1923 plates and 5 of the 1924

TABLE IV

ANNUAL MEANS OF CALCIUM (K) VELOCITY

JD 242+*	Velocity	No. of Plates	JD 242+	Velocity	No. of Plates
9907*...........	−23	1	6479...........	−15	5
3924...........	10	4	6866...........	12	3
4302...........	13	2	7235...........	13	9
4668...........	3	2	7581...........	−19	1
5784...........	13	9			
6142...........	− 9	6	Mean......	−12	....

* First three figures are 241 for first entry.

TABLE V

HYDROGEN-LINE WIDTHS AND EMISSION RATIO

JD 242+*	LINE WIDTHS IN ANGSTROMS			EMISSION RATIO
	$H\beta$	$H\gamma$	$H\delta$	
9914*........	3.93	3.00	....	$R>V$
3534........	4.16	2.78	2.82	$V\geqq R$
3924........	4.56	3.01	3.17	$R=V$
4302........	3.66	2.74	3.24	$R=V$
4668........	3.59	2.62	2.95	$R\gg V$
5471........	2.91	2.48	2.81	$R\geqq V$
5784........	3.72	2.68	2.82	$R\geqq V$
6142........	4.16	2.03	2.79	$R\geqq V$
6479........	4.72	3.05	2.76	$R>V$
6866........	4.41	3.13	2.73	$R=V$
7235........	4.75	3.13	3.01	$R=V$
7615........	3.56	2.56	2.10	$R=V$

* First three figures are 241 for first entry.

plates. In 1926, M. L. Zimmer measured all the plates taken in 1923, 1924, and 1925—a total of 26. The results are given in Table VI. Although the velocities from individual plates scatter more widely, it is reassuring to find the yearly means in substantial agreement.

All the writer's normal places are presented graphically in Figure 1. The emission ratios, and the weighted means of hydrogen, of

helium, and of iron are all represented by dots. The triangles stand for the weighted means of the helium triplets; the circles, for the

TABLE VI

RESULTS OF THREE MEASURERS

Year*	Hydrogen	Iron	Helium Triplets	Helium Singlets	All Helium
1923 (S)....	+6	+18	−13	+9	−2
(Z)....	+8	+23	11	−3	7
(C)...	+5	+19	16	+2	7
1924 (S)....	+7	+20	10	0	5
(Z)....	+3	+11	15	0	8
(C)...	+4	+16	18	+2	8
1925 (Z)....	0	+ 4	18	−6	12
(C)...	−2	− 4	−22	+3	− 9

* S, Smith; Z, Zimmer; C, Cleminshaw.

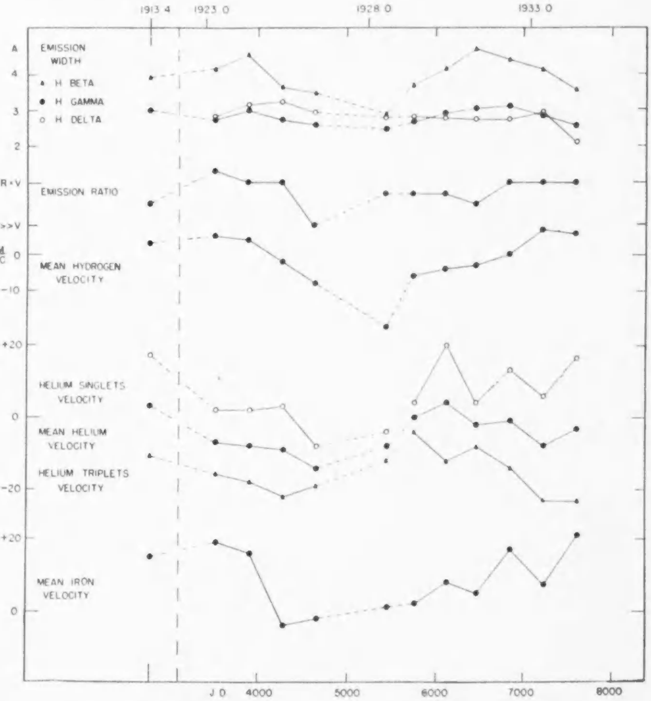

FIG. 1.—Changes in the emission lines and radial velocities of χ Ophiuchi

weighted means of the helium singlets. In the case of the line widths, $H\beta$ is represented by triangles, $H\gamma$ by dots, and $H\delta$ by circles. Connecting lines have been drawn as guides to the eye. The points at JD 5471 have been joined to the other points by broken lines, since they depend on only one plate in 1928. That plate was also measured by Dr. McLaughlin, and the adopted values are the means of his measures and the writer's. The 1913 results have been separated from the rest by a vertical broken line to emphasize the gap of ten years occurring there.

DISCUSSION OF RESULTS

Hydrogen.—There is no doubt of the variation of the hydrogen velocity. A period of about ten years is indicated, with maxima probably occurring in 1923 and 1933. The close agreement of the 1913 velocity with that in 1923 lends a little support to such a period. The 1896 Lick value of -11 km/sec is close to the 1926 value of -8 km/sec. However, the agreement of a few scattered points of possible earlier cycles is not very significant, and it must be understood that the suggested period is only tentative. It is unfortunate that no plates were obtained in 1927 and that only one was obtained in 1928, as a minimum was apparently reached during that interval. The one velocity during that time happens to be consistent with the rest of the curve.

The mean of all the hydrogen velocities is -1 km/sec. There appear to be variations of short period, and an unsuccessful attempt was made to determine this period. The changes are perhaps irregular.

The negative velocities are accompanied by a strengthening of the red emission edges relative to the violet. The 1928 value of the emission ratio should be lower, but these estimates are difficult to make, at best, and a single one is of little weight. The value at JD 6479 is also a little low, but the characteristic behavior of the Be stars is shown in the general correlation of the velocity-curve and the curve of the emission ratio.

The widths of the emission lines, $H\gamma$ and $H\delta$, appear to have been nearly constant, but the $H\beta$ curve looks like the velocity-curve, ex-

cept that the period is shorter. The means of all measures and their numbers are as follows:

Line	Width	Number of Measures
$H\beta$	4.12 A	107
$H\gamma$	2.87	105
$H\delta$	2.86	73

$H\gamma$ is of the same width as $H\delta$, which is unusual. However, these lines are narrow, and small differences of width are not easily detected. The narrowness of the lines may be attributed to a small inclination of the star's axis of rotation, as was suggested by Struve.[6] The total intensity of the emission has undergone no appreciable change.

Iron.—The velocity-curve of ionized iron is similar to that of hydrogen, though the variations do not occur simultaneously. The observations are not inconsistent with the suggested ten-year period, though the changes are not as regular as in the case of hydrogen. The mean of all iron velocities is +8 km/sec.

Helium.—The helium velocities are more irregular and are out of phase with those of hydrogen and iron. The most striking fact is the difference in the velocities of the singlets and the triplets. λ 4144 and λ 4388 belong to the same series, diffuse singlets. λ 4121 and λ 4472 are both triplets, though the first belongs to the sharp series and the second to the diffuse series. There is an average difference of 22 km/sec between the two pairs, the triplets having a mean velocity of −15 km/sec and the singlets +7 km/sec. The forms of the two curves are somewhat similar, but the curve for the singlets lags about one year behind the other.

A difference in behavior of these lines was found in β Monocerotis by Dr. M. Alberta Hawes.[7] The triplets and singlets were out of phase, but no mention was made of any difference in the mean velocities. M. K. Jessup, in an unpublished study of κ Draconis, found a difference in velocity of triplets and singlets in the same direction

[6] *Ap. J.*, **73**, 99, 1931.

[7] *Pub. Vassar Col. Obs.*, **4**, 36, 1934.

and of the same order of magnitude as in χ Ophiuchi. These three instances may indicate that such differences are characteristic of helium in the atmospheres of Be stars. One might well ask whether the cause is to be found in a different location in the atmosphere of the atoms producing the two sets of lines or in actual physical effects of environment upon the atoms in the states involved. Not even a guess can now be hazarded as to the answer to this question.

In this connection, it should be pointed out that Struve[8] found that the relative intensities of the helium lines are not the same in all stars. His results show that the ratio of intensity of singlet to triplet (4388/4472, 4009/4026, and 4144/4121) is greatest for stars of type B2 and decreases rapidly toward O and slowly toward B9. This behavior of the helium lines was also noted by Marshall.[9]

Calcium.—The K line of ionized calcium is of interstellar origin. The measures of it on 42 plates give a mean velocity of -12 km/sec. The angular distance of χ Ophiuchi from the solar apex at $18^{h}0^{m}$ and $+28°$ is $52°$. The component of the solar motion (19 km/sec) in the direction of this star is, therefore, -11.7 km/sec, in excellent agreement with the measured value.

CONCLUSION

χ Ophiuchi takes its place with the other Be spectrum variables in showing correlated changes of hydrogen velocity and emission ratio. It also has other points of resemblance to two Be stars. Agreement between the hydrogen and iron velocity-curves was found in 11 Camelopardalis;[10] also in that star and in β Piscium[11] the helium variations were out of phase with those of hydrogen. These two Be stars are like χ Ophiuchi in having single emission lines. It would seem to be a characteristic of Be stars for helium to be out of phase. The difference in velocity between the helium singlets and triplets should be looked for in other Be stars.

Flower Observatory of the University of Pennsylvania
Upper Darby, Pennsylvania
January 1936

[8] *Ap. J.*, **74**, 248, 1931.

[9] *Pub. U. of Michigan Obs.*, **5**, 162, 1934.

[10] *Pub. Vassar Col. Obs.*, **4**, 41, 1934.

[11] *Ibid.*, p. 53.

THE SPECTRAL VARIATIONS OF γ CASSIOPEIAE

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ABSTRACT

A study of 160 spectrograms of γ Cassiopeiae taken at the University of Michigan Observatory during an interval of 21 years (1914–1935) shows that its behavior is characteristic of the Be spectrum variables. The hydrogen lines show a correlation between changes of radial velocity and variations in the intensity ratio of the violet to the red emission components.

Observations from 1914 through 1928, like those made by R. H. Curtiss from 1911 to 1914, do not show any undoubted variation in velocity or emission ratio. Systematic changes began in 1929, resulting in a slow decrease in the hydrogen emission ratio and in the velocities of hydrogen and ionized iron. This was followed in 1932 by a more rapid increase, the violet component at the end of 1933 being more than twice as strong as the red, and the hydrogen velocity increasing by about 20 km/sec. During part of 1934 the lines were narrow and appeared single; and when both components were visible again at the end of 1934, the red one was nearly three times as strong as the violet, and there was a corresponding decrease of velocity.

The behavior of this star has been irregular, but it has been much like that of another Be star, β Monocerotis.

γ Cassiopeiae[1] was first observed spectroscopically in 1866, when Secchi noticed emission lines in its spectrum. A complete summary of the observations from that date until 1914 is given in the work of Curtiss.[2] Although the velocity was announced by Hartmann[3] as variable, the observations by Curtiss from 1911 to 1914 do not show any undoubted variation. The mean velocity obtained by him from 74 plates was -6.4 km/sec. The Lick observations[4] give a value of -4.7 km/sec. An emission velocity of -14.4 km/sec and an absorption velocity of $+8.5$ km/sec were found at the Yerkes Observatory.[5] The latter value, however, depends on only three plates. The velocity adopted in *Moore's General Catalogue of Radial Velocities*[6] is -6.8 km/sec.

Each of the hydrogen emission lines on the Michigan plates, except $H\beta$, is divided into two components by a central absorption line. In 1932, McLaughlin[7] suspected a slight change in the intensity ratio of the violet to the red emission component, especially from

[1] α $0^h50^m.7$, δ $+60°11'$, mag. 2.25, spectrum B0e.

[2] *Pub. U. of Michigan Obs.*, **2**, 1, 1916.

[3] *A.N.*, **173**, 102, 1906.

[4] *Pub. Lick Obs.*, **16**, 11, 1928.

[5] Frost, Barrett, and Struve, *Ap. J.*, **64**, 11, 31, 1926.

[6] *Pub. Lick Obs.*, **18**, 7, 1932.

[7] *Pub. U. of Michigan Obs.*, **4**, 177, 1932.

1915 to 1925. During the latter half of 1932, the violet component became much stronger than the red, and this led to the present attempt to determine whether this star, like other Be stars, has a variable radial velocity. The change in the emission ratio was found independently by Lockyer.[8]

THE OBSERVATIONS

This study is based upon 160 spectrograms of γ Cassiopeiae taken at the Observatory of the University of Michigan between October 24, 1914, and March 23, 1935. All of these were made with the one-prism spectrograph attached to the $37\frac{1}{2}$-inch reflecting telescope, with the exception of 16 plates which were taken with the two-prism spectrograph from August 21, 1927, to July 24, 1928, and one two-prism plate on January 27, 1935.

The hydrogen lines $H\beta$, $H\gamma$, and $H\delta$ were measured on almost all plates; and $H\epsilon$ and $H\zeta$ were measured on a few. $H\beta$ appears as a single emission line and three settings were made on it, one on each edge and one bisecting it. The bisection setting was averaged with the mean of the edge settings to give the adopted "velocity of the emission edges." The other hydrogen lines appear double, and five settings were made on each of them as follows:

1. Violet edge of violet emission component
2. Center of violet emission component
3. Center of central absorption component
4. Center of red emission component
5. Red edge of red emission component

From the mean of the edge settings was derived the velocity of the emission edges, and from the mean of the settings on the emission centers was found the velocity of the emission centers. The third setting gave the velocity of the central absorption. Emission widths were computed from the differences of the edge settings.

Measures were also made, on some of the plates, of several absorption lines of helium and of emission lines of ionized iron. However, these lines are so poor that the results were rejected, except in the case of the ionized iron lines, λ 4233 and λ 4584.

Visual estimates were made, by Dr. McLaughlin, of the ratios of intensity of the violet to the red component at $H\gamma$ and $H\delta$. This is

[8] *M.N.*, **93**, 362, 619, 1933.

referred to as V/R. He also estimated on many plates the ratio E/C, which is the intensity of the emission line relative to the neighboring continuous spectrum.

TABLE I

NORMAL PLACES OF VELOCITIES IN KM/SEC

JD 242+	Absorption			Emission Centers			Emission Edges				No. of Plates
	$H\gamma$	$H\delta$	Wtd. Mean	$H\gamma$	$H\delta$	Wtd. Mean	$H\beta$	$H\gamma$	$H\delta$	Wtd. Mean	
0447	−5	−10	−7	−6	−8	−7	−3	+4	−2	+1	4
0865	+13	−6	+7	+11	−8	+5	−6	−1	−15	−6	3
1150	−3	+1	−2	−1	.....	−1	−15	0	−15	−7	2
1596	−5	−10	−7	−7	−10	−8	−16	−9	−6	−10	2
2280	−10	0	−7	−9	+2	−5	−8	−7	−4	−7	9
2682	−1	−9	−4	+3	−6	0	−3	+2	−10	−2	4
2955	−7	−13	−9	−4	−12	−7	−11	0	0	−3	2
3726	+8	+6	+7	+2	+5	+3	+6	−1	0	+1	3
4108	−2	−20	−8	0	−15	−5	−2	−9	0	−5	2
4481	+3	−8	−1	+1	−4	−1	−2	−1	−5	−2	7
4850	−12	−5	−10	−11	−6	−9	−6	−8	−8	−8	15
4900	−3	−4	−3	+2	−4	0	−4	−8	−1	−5	12
5155	−5	−3	−4	−3	−4	−3	−4	−3	−4	−4	14
5492	−9	−6	−8	0	−5	−2	−14	−8	.....	−10	3
5860	−15	−20	−17	−14	−14	−14	−6	−14	−10	−11	6
6254	−16	−11	−14	−14	−4	−11	−12	−11	−15	−12	4
6639	.....	−8	−8	.....	−26	−26	0	−16	−20	−13	2
6912	−5	−23	−11	0	−15	−5	−17	0	−18	−9	2
6955	−10	−2	−7	−15	−9	−13	−7	−14	−3	−9	2
7007	+10	+7	+9	+15	+2	+11	+11	+8	0	+7	2
7028	−6	+11	0	−10	+8	−4	−11	−10	−2	−8	3
7045	−11	+19	−1	−4	+9	0	−11	−8	+3	−6	5
7126	−2	−8	−4	−2	−7	−4	(−14)	−2	−2	−2	5
7228	−3	0	−2	−7	−2	−5	(−25)	+3	+3	+3	6
7264	+7	−8	+2	+4	−4	+1	(−22)	−5	+2	−3	8
7284	+1	+4	+2	−2	+2	−1	......	−9	+4	−5	4
7345	+8	+22	+13	+9	+16	+11	......	+10	+25	+15	4
7382	+11	+15	+12	+2	+10	+5	......	+3	+14	+7	3
7478	+13	+18	+15	+1	+13	+5	(−29)	−9	+2	−5	9
7579	.....	.....	.....	.....	.....	.....	......	−15	−4	−11	2
7713	.....	.....	.....	.....	.....	.....	......	+5	+6	+5	6
7750	−34	−39	−36	−23	−26	−24	......	−6	−14	−9	2
7833	−6	−21	−11	−5	−15	−8	......	−7	−12	−9	3

RESULTS OF THE INVESTIGATION

The observations from the 160 spectrograms have been grouped into 33 normal places in Table I. This gives the velocities of the cen-

tral absorption and of the emission centers for $H\gamma$ and $H\delta$, and the velocity of the emission edges for $H\beta$, $H\gamma$, and $H\delta$. Weighted means are also included, $H\gamma$ having a weight of 2, and $H\beta$ and $H\delta$ each hav-

TABLE II

NORMAL PLACES OF EMISSION RATIOS AND EMISSION WIDTHS

JD 242+	Emission Ratio (V/R)			Emission Widths in A			No. of Plates
	$H\gamma$	$H\delta$	Wtd. Mean	$H\beta$	$H\gamma$	$H\delta$	
0447	1.02	1.00	1.01	4.63	4.24	3.84	4
0865	1.20	1.10	1.17	5.00	4.52	4.07	3
1150	1.20	1.00	1.13	5.22	3.81	3.49	2
1596	1.20	1.00	1.13	5.53	4.40	3.94	2
2280	1.00	1.07	1.02	5.34	4.28	4.24	9
2682	1.08	1.02	1.06	5.12	4.18	3.83	4
2955	0.80	1.00	0.87	5.31	4.10	4.22	2
3726	1.00	1.00	1.00	5.03	4.14	4.11	3
4108	1.15	1.00	1.10	5.38	4.60	4.53	2
4481	1.13	1.06	1.11	5.41	4.58	3.99	7
4850	1.05	1.03	1.04	5.91	4.58	4.62	15
4900	0.96	1.10	1.01	5.41	4.40	4.19	12
5155	1.01	1.03	1.02	4.96	3.95	3.87	14
5492	1.03	1.03	1.03	5.19	4.03	4.27	3
5860	0.95	1.05	0.98	5.44	4.40	4.24	6
6254	0.82	1.13	0.92	5.44	4.04	4.00	4
6639	0.85	0.85	0.85	6.09	4.32	3.49	2
6912	0.95	1.05	0.98	5.44	4.54	4.08	2
6955	1.05	1.30	1.13	5.69	4.86	4.13	2
7007	1.20	1.40	1.27	6.06	4.94	4.54	2
7028	1.30	1.40	1.33	6.69	5.29	4.93	3
7045	1.06	1.20	1.11	6.13	4.80	4.54	5
7126	1.48	1.58	1.51	5.22	4.62	4.34	5
7228	1.83	2.05	1.90	4.94	4.46	4.40	6
7264	2.18	2.03	2.13	6.03	4.60	4.21	8
7284	1.90	2.23	2.01	....	4.68	4.18	4
7345	2.50	2.70	2.57	....	4.46	4.43	4
7382	2.37	2.33	2.36	....	4.40	4.18	3
7478	2.30	1.95	2.18	4.47	3.71	3.79	9
7579	1.50	2.50	1.83	4.63	3.17	3.11	2
7713	1.00	1.00	1.00	3.31	3.03	2.57	6
7750	0.33	0.45	0.37	....	3.41	3.03	2
7833	0.38	0.39	0.38	....	3.50	3.22	3

ing a weight of 1. In the case of the velocity of the emission edges, the weighted means from JD 7126 to the end of the table depend on only $H\gamma$ and $H\delta$, and the values of $H\beta$ have been put in parentheses. The reason for this will be discussed later.

Table II contains the normal places of the emission ratio V/R and the emission widths. Weighted means of V/R are included, but no means of emission widths, since the values vary with wave-length.

The normal places are presented graphically in Figures 1 and 2. Figure 1 shows the line widths in angstroms, the emission ratio

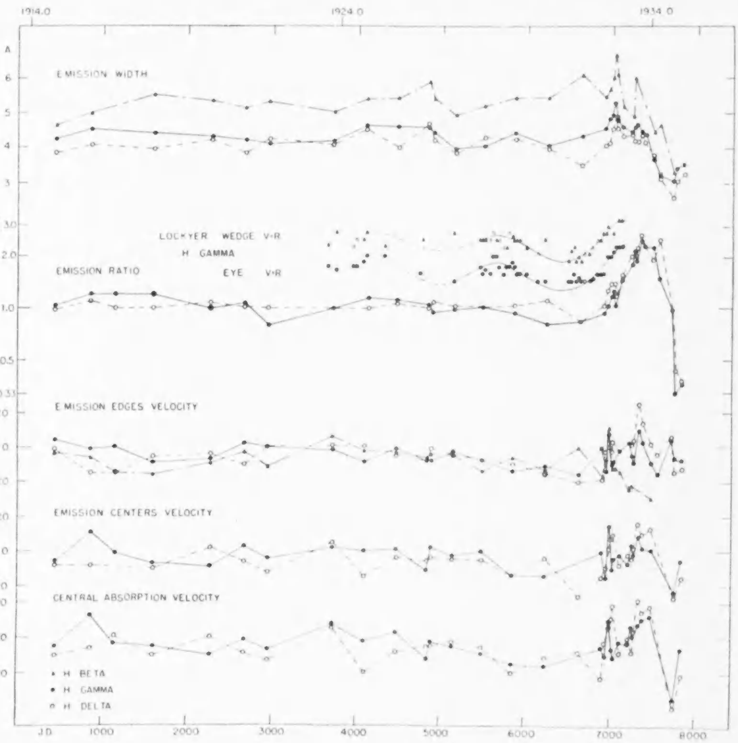

FIG. 1.—Line widths, emission ratios, and radial velocities in γ Cassiopeiae

V/R plotted logarithmically, and the velocities of the individual hydrogen lines. $H\beta$ is represented by triangles, $H\gamma$ by dots, and $H\delta$ by circles. To facilitate following the changes, the points have been connected by straight lines, which are not intended to represent the exact course of the variations.

For comparison, Lockyer's values of the emission ratio for $H\gamma$ have been reproduced from his paper,[9] in which he tentatively suggested

[9] *Ibid.*, p. 369.

a period of about four years. The filled triangles represent the "good," and the open triangles the "poor," wedge measures. The dots represent the "good," and the circles the "moderate," eye estimates. The curves have been inverted in order to agree in direction with the writer's curve, but no other change has been made in them. After Figure 1 had been completed, another paper by Lockyer appeared,[10] containing more observations, which have not been

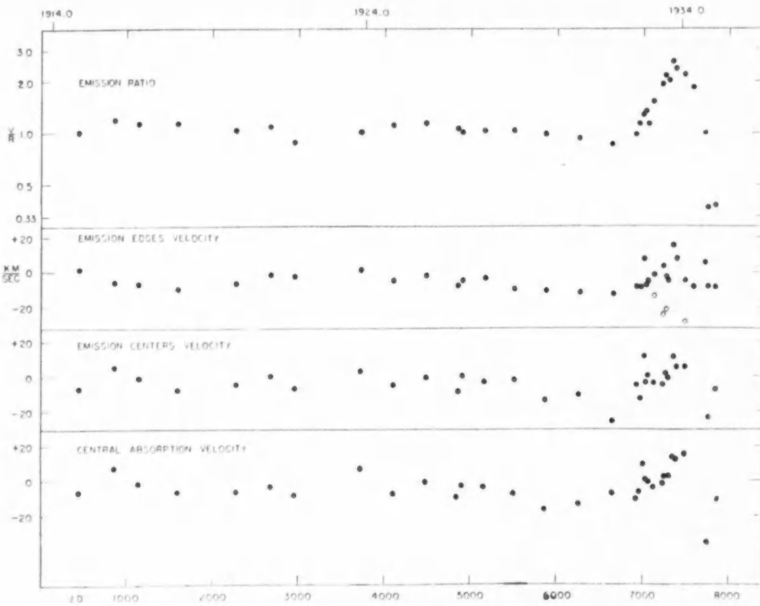

FIG. 2.—Means of hydrogen velocities and of emission ratios in γ Cassiopeiae

reproduced. In this later paper, he abandons the possibility of a four-year period.

Figure 2 represents the weighted means of the hydrogen velocity and of the emission ratio.

DISCUSSION OF RESULTS

In the discussion of the results, the normal places will be referred to by their Julian Day numbers, with the first three figures (242) omitted. First, it is of interest to examine the curves for a periodic variation. The most that can be said is that the velocity curves of

[10] *Ibid.*, **95**, 520, 1935.

Figure 2 show very slight indications of a period of about eleven years, with minima near JD 2000 and JD 6000. In this connection, four Yerkes observations[11] in 1918 give a mean emission velocity of -21 km/sec. This rather large negative value occurred during a season when no observations were made at Ann Arbor. The epoch for these four plates is JD 1811, near the time of the suggested possible minimum.

However, considering the interval from 1914 through 1928, including the normal place at JD 5492, one might well say that variations of velocity cannot be regarded as definitely established. In particular, there are only three points which depart conspicuously from the mean. Those at JD 0865 (second from the left) and at JD 3726 (middle of diagram) each depend on three plates, and each is influenced by a large positive value of one plate. The point at JD 4108 depends on only two plates, one of which is overexposed.

The weighted mean velocities of the 14 normal places in this interval (1914–1928) are -4.8 km/sec for the absorption, -3.4 km/sec for the emission centers, and -5.0 km/sec for the emission edges, with a probable error of ± 0.8 km/sec for the first two and ± 0.5 km/sec for the third. These are based on 82 plates and are very close to the value of -6.4 km/sec obtained by Curtiss for the years 1911–1914. The constancy of the velocity is confirmed by the emission ratio, which has a mean value of 1.04 for those 14 years. It dropped below unity only once, and it was never greater than 1.2 during that time. The conclusion is that the velocity and emission ratio remained constant from 1911 through 1928, except for possible short-period variations of very small amplitude.

Beginning in 1929, however, there is evidence of a systematic change. The normal place just preceding JD 6000 and the two following it are considerably lower than those in the first large group (1914–1928). The drop of the emission ratio below unity is confirmed by Lockyer's curves, which show the red component a little brighter than the violet. This is accompanied by drops in the velocity-curves, the change in absorption velocity slightly preceding the others.

In 1932 there is a very marked change in the opposite direction. All three emission-ratio-curves agree in showing that the violet com-

[11] *Ap. J.*, **64**, 31, 1926.

ponent becomes increasingly stronger than the red, and that the velocities become positive. There is an interruption in the rise of the curves; and for that reason the twelve normal places, from JD 6912 to JD 7478, inclusive, have been divided into two groups for comparison. The means of each group, together with the means of the two groups previously discussed, are given in Table III.

If the first group may be considered normal, the second is well below normal, the third slightly above, and the fourth well above. All three velocities and the emission ratio show this variation. The last value of the velocity of the emission edges would be lower if $H\beta$ were included in the mean. However, the four rather large negative values

TABLE III

MEANS OF FOUR GROUPS OF NORMAL PLACES

Date	Hydrogen					Iron			
	Abs.	Em. Centers	Em. Edges	V/R	No. of Plates	λ 4233	λ 4584	Mean	No. of Plates
1914–1928	− 4.8	− 3.4	− 5.0	1.04	82	− 8	− 7	− 8	24
1929–1931	−14.5	−15.0	−11.7	0.94	12	−25	− 2	−13	2
1932.9	− 2.3	− 2.4	− 4.5	1.25	19	+ 8	− 6	+ 1	10
1933.7	+ 6.9	+ 2.3	+ 0.3	2.16	34	+15	+18	+16	16

of $H\beta$, represented by open circles in Figure 2, are believed to be erroneous, owing to the shift of the center of density of the unresolved double line toward the violet. $H\beta$ is not resolved on the plates, and the increase in the strength of the violet part relative to the red probably causes one to make settings systematically in error in the negative direction.

The velocities derived from absorption, emission centers, and emission edges are in close agreement within each of the first three groups in Table III. In the fourth group, the absorption velocity is about 6 km/sec higher than the mean of the emission velocities. However, it is uncertain whether this difference is real. The red component during this interval was so weak on many of the plates that settings on the center and on the edge of it were difficult to make.

Also included in Table III are the results of the measures of the ionized iron lines, λ 4233 and λ 4584. The values of individual plates

scatter considerably, but they have been divided into the same four intervals of time as the hydrogen lines, and means have been taken. From these it appears that iron undergoes changes similar to those of hydrogen, but possibly of greater amplitude.

In May, 1934, a significant change was noted in the hydrogen lines. $H\gamma$ and $H\delta$, which had always been double emission lines, appeared single on May 21 (J.D. 7579). The central absorption was missing, and it did not appear again until the following November. Unfortunately, no plates were taken during the summer, but six plates in September and October show the lines as single.

This confirms the work of Lockyer,[12] who found that "$H\gamma$, $H\delta$ and $H\epsilon$, during the long period from 1934 March 27 to November 2 exhibited only one strong bright line." His interpretation was that "during those months this bright line, which was the violet component in 1934 February, maintained its intensity, but gradually moved towards the red end of the spectrum, and just at the end of this period, namely, 1934 December 20, reached the position where the red component should appear." He reached this conclusion by placing the negatives film to film and noting a distinct creep of the line from its initial position opposite the violet component (on a negative showing both components) to its final position opposite the red component.

It will be of interest to see if such a change is verified by the measures of the Ann Arbor spectrograms. All the plates from 1914 through 1926, during which interval the velocity and the emission ratio appeared to be constant, were used to find what might be called the "normal" positions of the emission components. For $H\gamma$, the results, in km/sec, were -77 for the violet component and $+75$ for the red. For $H\delta$, the values were, respectively, -88 and $+78$. The two plates of May 21, 1934, give a weighted mean velocity of -11 km/sec for the single emission lines, $H\gamma$ and $H\delta$. The corresponding value for the plates from September 25 to October 13 is $+5$ km/sec. From this it is seen that the center of the single emission line was not near the normal position of either the violet or the red components, and that it shifted only 16 km/sec, or about one-tenth the distance separating them.

From the 1914 to 1926 measures, the doublet separation in km/sec

[12] *M.N.*, **95**, 525, 1935.

is 152 for $H\gamma$ and 166 for $H\delta$. From November 7, 1934, to March 23, 1935, both lines have a separation of 116 km/sec. This decrease has been previously noted by J. F. Heard[13] on several Yerkes plates of higher dispersion. One of these occurred during the time when the emission was single on the Ann Arbor plates. On September 21, 1934, a doublet separation of 113 km/sec is given for the two *Fe* II lines, λ 5169 and λ 5316. This suggests that possibly the reason for the appearance of the lines as single on Lockyer's objective-prism plates and on the Ann Arbor one-prism plates is their limited resolution.

A possible interpretation of what happened is that the strong violet component and the weak red one approached each other until they were unresolved, then became equal in intensity, then reversed their inequality, and finally moved apart again. For the separation of the components to be smallest when they are equal is typical of several Be spectrum variables, notably 25 Orionis[14] and φ Persei.[15] With this interpretation, γ Cassiopeiae has done what other Be stars do, and its behavior seems unusual only because of the more conspicuous change in the width of the lines.

There appears to have been no definite variation in the widths of the emission lines until 1932. At that time there was a marked increase in all three line widths, which reached their maxima when the emission ratio reached its first peak. Then they decreased while the emission ratio went up to its maximum value, though one point on the $H\beta$ curve seems to indicate that it reached a secondary peak. The widths continued their decrease until October, 1934, when they reached minimum values of 3.31 A for $H\beta$, 3.03 A for $H\gamma$, and 2.57 A for $H\delta$. These may be contrasted with the mean widths of the lines, as found by Curtiss[16] and by the writer, as follows:

	$H\beta$	$H\gamma$	$H\delta$
1911–1914.....	5.06 A	4.40 A	3.94 A
1914–1931.....	5.35	4.30	4.14

[13] *Ap. J.*, **81**, 341, 1935.

[14] Helen W. Dodson, dissertation, University of Michigan, unpublished.

[15] H. F. Schiefer, dissertation, University of Michigan, unpublished.

[16] *Pub. U. of Michigan Obs.*, **2**, 1, 1916.

The ratio of the emission to the continuous spectrum does not show definite variations. When the components are very unequal, there is an appearance of a change; but this is probably not real, for the weakening of one component nearly, if not quite, balances the strengthening of the other. The definite determination of changes of intensity of emission would require measures on a microphotometer, of plates of higher dispersion than those available.

The behavior of this star is characteristic of the Be spectrum variables, as is shown by the correlation of velocities with emission changes. Though the variations appear to be irregular, a striking similarity is shown in the curves of the hydrogen velocities and of the emission-ratios of β Monocerotis, which was studied by Dr. Hawes.[17] In that case, the emission ratio, V/R, of $H\beta$ remained

TABLE IV

	Abs. Vel.	Em. Vel.	Log V/R
	(km/sec)	(km/sec)	
π Aquarii	120	90	0.78
25 Orionis	130	110	.64
β Monocerotis	50	75	.60
γ Cassiopeiae	47	32	0.88

equal to unity for about 15 years, then dropped below unity for several years, and finally increased rather suddenly to a value greater than 2. Corresponding changes occurred in the velocity, though it decreased sooner than did V/R. On the basis of the mechanism of Be stars suggested by McLaughlin,[18] it would appear that in the cases of both γ Cassiopeiae and β Monocerotis, the stellar atmosphere remained quiescent for a considerable time, then slowly expanded, and then rapidly collapsed. The changes in γ Cassiopeiae during 1934 would indicate another expansion.

The range in velocity is not as great as might be expected from the changes in the emission ratio. Table IV gives a comparison of the ranges, in round numbers, of absorption and emission velocities and

[17] *Pub. Vassar Coll. Obs.*, **4**, 36, 1934.

[18] *Proc. Nat. Acad.*, **19**, 44, 1933.

of log V/R of $H\gamma$ in π Aquarii,[19] 25 Orionis,[20] β Monocerotis,[21] and γ Cassiopeiae. Of these four stars, γ Cassiopeiae has the smallest range of velocity and the largest range of emission ratio.

I wish to thank Dr. Heber D. Curtis for the equipment and observational material made available to me; Dr. Dean B. McLaughlin for his very helpful suggestions and advice; and the Executive Board of the Graduate School of the University of Michigan for the grant of a university fellowship which made this study possible.

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[19] *Pub. U. of Michigan Obs.*, **4**, 44, 1931.
[20] H. W. Dodson, *op. cit.*
[21] *Pub. Vassar Coll. Obs.*, **4**, 31, 1934.

REVIEWS

The Theory of Atomic Spectra. By E. U. Condon and G. H. Shortley. New York: Macmillan Co., 1935. Pp. 441. $11.00.

The Theory of Atomic Spectra, by E. U. Condon and G. H. Shortley, does not pretend to be an elementary introduction to the subject. The authors specifically state that they have not aimed at any complete formulation of the principles of quantum mechanics, since several elementary textbooks are available.

The first chapter of the book is introductory and historical. The most difficult and abstruse sections of the book are to be found in chapter ii, an exposition of the quantum-mechanical methods employed by the authors. The difficulty lies in the fact that the theoretical approach is via the matrix algebra rather than the more readily visualized wave mechanics. The Dirac formulation, while admittedly elegant, is not as familiar to the average reader as the Schrödinger representation.

The remainder of the volume is devoted to the two aspects of atomic phenomena that are of chief concern to the astrophysicist, viz., energy levels and the intensities of spectral lines. The book is unquestionably the most complete theoretical discussion now available of the spectra of both simple and complex atoms. The results of the various investigations are of extreme importance to the astrophysicist, and numerous astronomical problems are discussed, such as forbidden lines and absorption coefficients. An extremely useful and complete treatment of the hydrogen-intensity problem is given. The questions of absolute intensities and transition probabilities are discussed in considerable detail. It is unfortunate that the abstruse character of the book is likely to limit its usefulness to astrophysicists, for the subject matter dealt with is fundamental to anyone who is working on the interpretation of absorption and emission lines.

Donald H. Menzel

Lunettes et télescopes. By A. Danjon and A. Couder. Editions de la *Revue d'optique théorique et instrumentale*. Paris, 1935. Pp. 715. Unbound, Fr. 100; bound, Fr. 120.

The authors have given a clear and very complete account of the theory, adjustment, and use of telescopes and of many kinds of telescopic equipment. Included in the contents, in addition to the more

common topics, are discussions of the mirror systems of Schwarzschild, Chrétien, and Schmidt, and a seventy-five-page section dealing with aberrations of lenses and mirrors. The subjects of telescope objectives and mirrors are treated at considerable length and more than one-third of the book is devoted to the construction of instruments. The five general divisions of the work are: general theory and conditions of use; description of aberrations; objectives, mirrors, and eyepieces; the construction of instruments; and a historical section. In a work of such length it is unfortunate that instruments of the importance of the spectroheliograph and the spectrohelioscope are not mentioned, and the section on spectroscopy is certainly not as complete as might be expected. The book should, nevertheless, be of considerable value to the practical astronomer.

W. W. MORGAN

Il sole. By G. ABETTI. Edition Ulrico Hoepli. Milano, 1936. Pp. 410. Unbound, L. 22.

The present work, which is a revision and enlargement of the author's monograph in the *Handbuch der Astrophysik,* is a very readable and clearly written description of the present status of our knowledge of the sun. The numerous illustrations are of excellent quality.

ERRATUM

In Volume **81**, page 174, of the *Astrophysical Journal* the ordinates of the lower part of Figure 1 should read +0.120, +0.140, +0.160, +0.180, instead of +1.20, +1.40, +1.60, and +1.80.

C. T. ELVEY

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SUBJECTS

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AUTHORS